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A quasi-one-dimensional model of thermoacoustics in the presence of mean flow

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ABSTRACT

In thermoacoustic regenerators, the interaction of thermo-viscous boundary layers and axial temperature gradients causes a conversion from thermal energy to acoustic power or vice versa. In this paper, an improved analytical model for thermoacoustic boundary layer effects in the presence of mean flow is derived and analyzed. Previous formulations of the thermo-acoustic effect take into account effects of mean flow on acoustic propagation only implicitly, i.e. in as much as mean flow influences the mean temperature field. The new model, however, includes additional terms in the perturbation equations, which describe explicitly the interaction between steady mean flow and acoustics. For a parallel plate pore the three-dimensional thermoacoustic equations are derived and reduced to a transversally averaged system of differential equations by applying Green's function technique and suitable assumptions. The resulting one-dimensional perturbation equations are then solved numerically for two sets of boundary conditions to obtain the linear scattering matrix coefficients. The solutions, generated for a wide range of frequencies, can be applied in a low-order “network model” context to study the stability of thermoacoustic devices. The impact of mean flow on the thermoacoustic interaction is investigated and validated against full computational fluid dynamics simulations of laminar, compressible flow for one specific configuration. It is shown that at low frequencies (Womersley number < 1) the new formulation predicts the acoustic behavior more accurately than the earlier formulations. Finally, the ideas and benefit of further improved and more complex models for higher Mach numbers are discussed.

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1. Introduction

The interest in the interaction of acoustics and heat flow dates back more than 200 years [1,2]. One of the investigated issues is the thermoacoustic (TA) effect: in narrow geometries thermo-viscous boundary layers interact with axial mean temperature gradients. This interaction causes a conversion of thermal energy to acoustic power or vice versa. Sondhauss [3] observed this effect first. He described the occurrence of spontaneous loud sound emissions in narrow pipes during the glass blowing process. Although Rayleigh [4] had already given the first qualitative explanations for this effect, only incomplete descriptions of the phenomenon were provided by different authors [5,6]. Rott was the first author to completely describe

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Nomenclature

b	inhomogeneous term
c	speed of sound, m s^{-1}
c_p	specific heat at constant pressure, $\text{J kg}^{-1} \text{K}^{-1}$
c_v	specific heat at constant volume, $\text{J kg}^{-1} \text{K}^{-1}$
$\tilde{\mathfrak{A}}$	characteristic wave amplitude, m s^{-1}
$\mathfrak{G}$	characteristic wave amplitude, m s^{-1}
h	specific enthalpy, J kg^{-1}
k	body force, $\text{m}^2 \text{s}^{-2}$
K	thermal conductivity, $\text{W m}^{-1} \text{K}^{-1}$
L	length, m
La	Lautrec number
Ma	Mach number
Ma_a	acoustic Mach number
p	pressure, Pa
Pe	Péclet number
Pr	Prandtl number
R	radius, m
Re	Reynolds number
U	flow velocity, ms^{-1}
Wo	Womersley number
$\mathfrak{R}$	gas constant, $\text{J kg}^{-1} \text{K}^{-1}$
t	time, s
T	temperature, K
u	velocity, m s^{-1}

Greek characters

β	compressibility, K^{-1}
γ	heat capacity ratio
δ	boundary layer thickness, m
ϵ_S	influence factor of solid–fluid interaction
ϵ	ratio of dimensions
η	ratio of dimensionless quantities
ϑ	transversal distribution
κ	Helmholtz number
μ	dynamic viscosity, $\text{kg m}^{-1} \text{s}^{-1}$
ν	kinematic viscosity, $\text{m}^2 \text{s}^{-1}$
ρ	density, kg m^{-3}
σ	thermal conductivities ratio
τ	shear stress tensor, N m^{-3}

Φ	blockage ratio
ψ	generic variable, axial distribution
ω	angular frequency, rad s^{-1}

Subscripts and abbreviations

0	mean quantity
1	acoustic quantity
a	acoustic
m	boundary
d	downstream
u	upstream
lin	linear
exp	exponential
BC	boundary condition
CFD	computational fluid dynamics
CFL	Courant–Friedrichs–Lewy
$LODI$	local one-dimensional inviscid
LES	large eddy simulation
$MISO$	multiple input single output
SI	system identifications
$SISO$	single input single output
GF	Green's function
ODE	ordinary differential equation
Λ	modeled term
ζ	concerning bulk viscosity
IMP	implicit model
K	thermal
M I	model I
M II	model II
ν	viscous
ref	reference
S	solid
TA	thermoacoustic
E	energy conservation
G	gas law
$\overline{G}$	averaged gas law
$\overline{k}$	intermediate averaged continuity
$\overline{K}$	final averaged continuity
T	energy conservation
$\times$	axial momentum conservation

the phenomenon analytically in a series of publications [7–12]. Wheatley [13], his successor Swift [14] and finally in't panhuis [15] and in't panhuis et al. [16] improved the analytical understanding of the TA effect. Arnott et al. [17] enhanced the applicability of this approach for arbitrary pore shapes. Watanabe et al. [18] further improved these models by accounting for nonlinear effects such as drag or heat transfer. The TA effect has also been investigated in various computational fluid dynamics (CFD) simulations (e.g. [19–23]). Their analytical formulations facilitate the description of the axial propagation of acoustic velocity u_1 and pressure p_1 in terms of their cross-section averaged quantities. These transversally averaged parameters $\langle p_1 \rangle$, $\langle u_1 \rangle$ form a set of transport equations in the axial pore direction. Those transport equations are applied in a one-dimensional numerical tool [24] to predict the operating conditions in TA devices.

Previous publications have presented ideas for probable applications of TA devices [13,14], and some authors claim that the technology is ripe for the market [25]. Nevertheless, almost 30 years after the first attempts of technical realizations of such apparatus, the technology is only applied in niche applications [26–28]. Thus, the main aim of researchers remains to expand the applicability of TA for cooling and power generation.

All cited authors restrict their descriptions to quiescent mean flow conditions. Until now, only a few publications have tried to deal with mean flow affected TA power conversion [29–31]. Bammann et al. [32] provide an overview over the published literature on flow-through TA refrigeration. One of the main reasons for the modest interest in their topic is the

lack of proper modeling of the TA interaction effects under such conditions. All hitherto referenced publications consider the impact of mean flow only implicitly by adapting the axial mean temperature profile, or do not address it at all. Based on the assumption that the mean flow is due to streaming effects induced by acoustic quantities, a second-order enthalpy flux consideration leads to a distortion of the mean temperature profile [8,33–35]. The present paper presents and analyzes an improved quasi-one-dimensional method, which explicitly takes the laminar mean flow into account in the perturbation equations. Using this approach enables the prediction of the performance of TA devices that exploit mean flow as a positive effect, e.g. Reid's refrigerator [30]. Due to the resulting one-dimensional form, the computational effort for the calculation of one configuration is comparatively small. Conceivable applications may, for example, arise in exhaust gas streams, where the waste heat enthalpy flux is directly transformed into acoustic power.

The derivation of the quasi-one-dimensional model is based on the method applied by in't panhuis et al. [16,15], who derived TA transport equations that account for almost arbitrary stack pore geometry. Like all other proposed approaches, the model takes into account the so-called long-pore assumption, i.e. the hydraulic diameter $2R$ is much smaller than the characteristic length in axial direction L . Further, in't panhuis [15] follows the common approach of neglecting the contribution from mean flow when linearizing the basic set of equations. In the present work, this assumption is not invoked and mean flow is explicitly incorporated.

Taking into account the contribution of mean flow leads to an increase in the number of terms in the considered equations. Hence, for the purpose of demonstrating the impact of explicitly accounting for mean flow, the solution is derived for a generic problem resulting in the least complex system of transport equations. The geometrical conditions and thermo-physical correlations are chosen to keep the equations short. Sticking to the general solution with only a few necessary assumptions is still feasible; however, this complicates the interpretation of the impact of mean flow due to the huge numbers of terms inside the system of equations. Thus, to simplify the model where possible, assumptions will be introduced. These assumptions due to compactness of equations are marked as such.

Apart from spatial distributions of the acoustic quantities for a single frequency, acoustic scattering matrices are used for validating the one-dimensional models against CFD results. The latter validation tool is mostly used to investigate the stability of the system in the linear regime by using 1D network models [36]. Although TA engines generally operate in nonlinear limit cycles, the identified growth rate at the onset of the acoustic instability is a quantitative measure for the achievable pressure amplitudes. Altogether, as the simplicity of the solution is a central request, this publication is restricted to the linear acoustic regime. If nonlinearities like the acoustic feedback into the mean temperature distribution or saturation effects have to be taken into account, all steps can be processed for the nonlinear situation.

The paper is organized as follows: In Section 2, the geometrical and mean flow conditions are presented. In Sections 3 and 4, the one-dimensional acoustic transport equations are derived for a parallel plate stack pore. The flow chart of the derivation is sketched in Fig. 1: First the compressible Navier–Stokes equations (NSE) are non-dimensionalized using narrow geometry assumptions. These dimensionless NSEs experience a simplified modal expansion. The mean flow field is solved analytically from the resulting zeroth-order set of equations. These solutions are used as input to the first-order set of equations, the linearized dimensionless NSEs. This set of equations for the acoustic field is then solved in segregated steps: First Green's function (GF) solution technique is applied on the axial momentum and energy equations. This yields formulations for their transversal shapes. Closure assumptions have to be made for two convective terms to obtain an analytical solution of these formulations. Integrating the resulting equations in the transversal direction creates a set of averaged perturbation equations. Finally, eliminating density and temperature perturbations leads to one-dimensional TA transport equations in terms of velocity and pressure oscillations. The resulting set of equations accounts for non-zero mean flow contributions, which appear explicitly in some terms. Thus, models based on this system of equations are named “explicit” in the rest of the paper. The existing model [37] is denoted as “implicit” (IMP), because in that model mean flow influences the system of equations only implicitly through its impact on the mean temperature profile, which is computed from a second-order energy equation that considers enthalpy transport by the mean flow [8]. For the explicit equations two

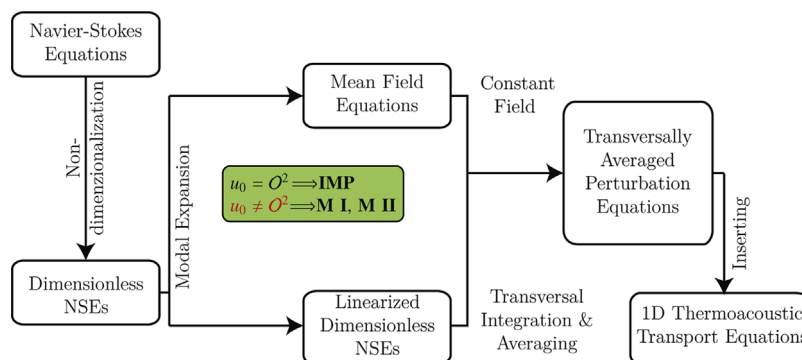


Fig. 1. Derivation adapted from in't panhuis et al. [16], which considers mean flow only implicitly (IMP). In this approach, the mean velocity is explicitly taken into account. Model approximations **M I**, **M II** must be invoked to obtain analytical solutions.

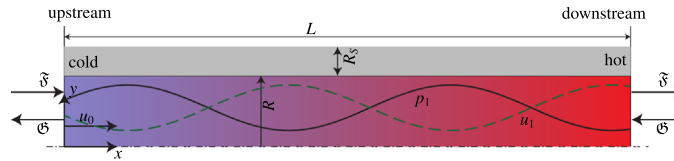


Fig. 2. Sketch of the observed problem: The acoustics in a slab pore, i.e. a narrow channel of length L and constant hydraulic radius R within a non-isothermal axial field and mean flow are contemplated. The sinusoidal lines indicate the profile of the acoustic quantities p_1 and u_1 . These quantities can be transformed into characteristic wave amplitudes δ, ϕ . They form the base for the scattering matrix notation which correlates these incoming and outgoing waves at both ends of the channel. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this paper.)

alternative approaches for the necessary closure assumptions are investigated. Prediction accuracy of these two approaches, denoted by **M I** and **M II**, is tested for the spatial acoustic propagation for a single frequency and scattering behavior for different test cases against a full CFD simulation and the existing implicit one-dimensional solution method. Finally, an outlook to improved modeling approaches is presented.

2. Problem description

2.1. Geometric settings

A channel of length L and constant hydraulic radius R is one of the most simple geometries to be considered for a TA system. Restricting the problem to a constant pore height simplifies the contact conditions of the fluid and solid regions in the problem. Choosing a channel geometry leads to a two-dimensional problem in space, which enormously reduces the size of the derived equations. The solid plates in the problem sketched in Fig. 2 are $2R_s$ thick. The origin of the Cartesian coordinate system is positioned at the upstream (u) end in the center of the channel. The counter side of the pore at $x=L$ is indicated as the downstream (d) position. Note that any quantity featuring an index S refers to the solid stack.

2.2. Thermo-physical conditions and mean parameter assumptions

The solid region inside the pore is assumed to consist of a homogeneous material. All solid properties are only functions of the mean temperature. The walls are non-permeable for the working fluid, which is assumed to be an ideal gas. The viscosity ν , thermal conductivity K and the specific heat capacities c_p, c_v are assumed to be functions of mean pressure p_0 and temperature T_0 . The index 0 refers to (temporal) mean quantities, while an index 1 denotes a time dependent perturbation quantity.

The blue to red colorized gradient in the lower part of Fig. 2 indicates the temperature distribution inside the pore. Here, the blue indicates the cold side, whereas the hot downstream side is marked in red.

For the purpose of comparison with experiments, the terms upstream and downstream, which indicate a mean flow from the cold to the hot side are introduced. In this paper all reference parameters are chosen to be the cold upstream values. The mean flow is presumed to be laminar. Although the incompressibility assumption (see [38]) might be violated by the strong impact of the axial temperature, a Poiseuille-like mean flow field is expected.

3. Non-dimensional, linearized Navier–Stokes equations of a slab pore

This section covers the first four steps of the sketch depicted in Fig. 1: At first the Navier–Stokes equations are non-dimensionalized. The resulting set of equations is then split into transport equations describing the mean flow field and the propagation of acoustic quantities. Here, a special modal expansion technique is applied.

As boundary layer effects are the cause of the TA effect, the Navier–Stokes equations are a suitable choice of equations describing the motion of the fluid under consideration. The equations in the notation of Landau and Lifshitz [39] read as

$$\frac{D\rho}{Dt} = -\rho \nabla \cdot \mathbf{v} \quad (1)$$

$$\rho \frac{D\mathbf{v}}{Dt} = -\nabla p + \nabla \cdot \boldsymbol{\tau} + \rho \mathbf{k} \quad (2)$$

$$\rho \frac{Dh}{Dt} = \beta T \frac{Dp}{Dt} + \nabla \cdot (K \nabla T) + \boldsymbol{\tau} : \nabla \mathbf{v}. \quad (3)$$

where

$$\frac{D\Psi}{Dt} = \frac{\partial \Psi}{\partial t} + \mathbf{v} \cdot \frac{\partial \Psi}{\partial \mathbf{x}}$$

denotes the substantial derivative of a transported variable Ψ . The stress tensor τ of a Newtonian fluid is described by

$$\tau = 2\mu \frac{1}{2} (\nabla \mathbf{v} + (\nabla \mathbf{v})^T) + \zeta (\nabla \cdot \mathbf{v}) \mathbf{I}.$$

As the body forces $\mathbf{k}$ are negligible in the inspected problem, the corresponding term can be omitted. The set of differential equations is closed using an equation of state relating density and pressure. Since, in this case, the ideal gas law

$$p = \rho \Re T \quad (4)$$

is used, the transport of enthalpy h – cf. Eq. (3) – can be rewritten in terms of temperature T

$$\rho c_p \frac{DT}{Dt} = \beta T \frac{Dp}{Dt} + \nabla \cdot (K \nabla T) + \tau : \nabla \mathbf{v}. \quad (5)$$

3.1. Non-dimensionalizing the NSEs

The system of equations (1)–(5), (the NSEs) are non-dimensionalized with respect to the characteristic scales of the geometry under consideration. Olson and Swift [40] first derived a set of dimensionless parameters describing TA phenomena by applying the Buckingham Π theorem. Here, we proceed in the same manner as in't panhuis et al. [16] for a two-dimensional set of equations formulated in Cartesian coordinates. The corresponding dimensionless quantities are as follows:

$$\begin{aligned} x &= L\tilde{x}, \quad y = R\tilde{y}, \quad t = \frac{L}{c_{\text{ref}}}\tilde{t}, \quad u = c_{\text{ref}}\tilde{u}, \quad v = \varepsilon c_{\text{ref}}\tilde{v} \\ q &= q_{\text{ref}}\tilde{q}, \quad p = q_{\text{ref}}c_{\text{ref}}^2\tilde{p}, \quad T = \frac{c_{\text{ref}}^2}{c_{p,\text{ref}}}\tilde{T}, \quad c = c_{\text{ref}}\tilde{c}, \quad \beta = \frac{c_{p,\text{ref}}}{c_{\text{ref}}^2}\tilde{\beta} \\ K &= K_{\text{ref}}\tilde{K}, \quad c_p = c_{p,\text{ref}}\tilde{c}_p, \quad \tau = \frac{\mu_{\text{ref}}c_{\text{ref}}}{R}\tilde{\tau}, \quad \mu = \mu_{\text{ref}}\tilde{\mu}, \quad \zeta = \zeta_{\text{ref}}\tilde{\zeta} \end{aligned}$$

using the index ref for the reference parameters. All axial terms are non-dimensionalized using the length L of the pore, while for all transversal terms, v, y and the shear tensor τ , the hydraulic radius R of the pore is used. The resulting dimensionless numbers are listed in Table 1. The viscous (ν), the volume viscous (ζ) and thermal (K for gas, S for solid) penetration depths δ are defined as follows:

$$\delta_\nu = \sqrt{\frac{2\mu_{\text{ref}}}{\omega q_{\text{ref}}}}, \quad \delta_\zeta = \sqrt{\frac{2\zeta_{\text{ref}}}{\omega q_{\text{ref}}}}, \quad \delta_K = \sqrt{\frac{2K_{\text{ref}}}{\omega q_{\text{ref}}c_{p,\text{ref}}}}, \quad \delta_S = \sqrt{\frac{2K_S}{\omega q_S c_S}}. \quad (6)$$

Table 1
“ Π -Group” for thermoacoustics in narrow pores.

Symbol	Formula	Description
γ	$\frac{c_p}{c_v}$	Heat capacity ratio
ε	$\frac{R}{L}$	Ratio of dimensions
σ	$\frac{K_S}{K_{\text{ref}}}$	Thermal conductivities ratio
ϕ	$\frac{A_{\text{gas}}}{A}$	Blockage ratio
κ	$\frac{\omega L}{c}$	Helmholtz number
Ma_a	$\frac{U}{c}$	Acoustic Mach number
Fr	$\frac{U^2}{gL}$	Froude number
La	$\frac{R}{\delta_K}$	Fluid Lautrec number
La_S	$\frac{R}{\delta_S}$	Stack Lautrec number
Pr	$\frac{\delta_\nu^2}{\delta_K^2}$	Prandtl number
Wo	$\frac{R}{\delta_\nu}$	Womersley number based on δ_ν
Wo_ζ	$\frac{R}{\delta_\zeta}$	Womersley number based on δ_ζ

In dimensionless form, the governing equations now read as

$$p = \frac{\gamma - 1}{\gamma} T \varrho \quad (7a)$$

$$\frac{D\varrho}{Dt} = -\varrho \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \quad (7b)$$

$$\varrho \frac{Du}{Dt} = -\frac{\partial p}{\partial x} + \frac{\kappa}{Wo^2} \frac{\partial^2 u}{\partial y^2} + \varepsilon \left[\frac{\kappa}{Wo_\zeta^2} \left(\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 u}{\partial x \partial y} \right) + \frac{3\kappa}{Wo^2} \frac{\partial^2 u}{\partial x \partial y} \right] + \mathcal{O}(\varepsilon^2) \quad (7c)$$

$$0 = -\frac{\partial p}{\partial y} + \varepsilon \frac{\kappa}{Wo^2} \frac{\partial^2 u}{\partial y^2} + \mathcal{O}(\varepsilon^2) \quad (7d)$$

$$\varrho \frac{DT}{Dt} = \beta T \frac{Dp}{Dt} + \frac{\kappa}{2La^2} \frac{\partial^2 T}{\partial y^2} + \mathcal{O}(\varepsilon^2) \quad (7e)$$

$$\varrho_s \frac{\partial T}{\partial t} = \frac{\kappa}{2La_s^2} \frac{\partial^2 T}{\partial y^2} + \mathcal{O}(\varepsilon^2). \quad (7f)$$

From here on, the tilde, characterizing dimensionless variables, is omitted for reasons of clarity and readability. Here, the transversal momentum equation (7d) is multiplied by the square of the ratio of dimensions $\varepsilon = R/L$. Hence, the smallest non-vanishing term contains information of $\mathcal{O}(\varepsilon^0)$. In the equation of state (7a) T , p and ϱ correlate with the specific heat capacity ratio $\gamma = c_p/c_v$. The Womersley numbers Wo and Wo_ζ denote the ratio of hydraulic radius to the viscous boundary layer thicknesses δ_v and δ_ζ respectively. Their thermal equivalent are the solid (s) and fluid Lautrec numbers La . La and Wo scale in terms of the square root of the Prandtl number, i.e. $Pr = \delta_v^2/\delta_\kappa^2$. Although La is generally used to characterize TA devices by their porous media [41], the authors use the Wo notation because of its familiarity from aero-acoustic applications. The frequency dependent component of the Helmholtz number $\kappa = \omega L/c$ in the viscous and thermal terms is balanced by the square of one of these four dimensionless numbers. The set of transport equations (7f) describes the full fluid dynamics of a perfect gas inside a narrow pore at low Ma in a non-dimensional manner.

3.2. Separating acoustics by modal expansion

In the next step, the acoustic perturbations of the flow field variables are separated from their mean values. Here, as the mean flow velocity is related to the speed of sound c , a special form of modal expansion of the flow variables in terms of the acoustic (index a) Mach number $Ma_a = u_1/c_{\text{ref}}$ is applied. This procedure assumes that any variable $\Psi(\mathbf{x}, t)$ may be split into a temporally constant mean quantity $\Psi_0(\mathbf{x})$ and a harmonically oscillating part $\Psi_1(\mathbf{x})\mathbf{e}^{ikt}$. The full standard expansion then reads as

$$\Psi(\mathbf{x}, t) = \Psi_0(\mathbf{x}) + Ma_a \Psi_1(\mathbf{x})\mathbf{e}^{ikt} + \mathcal{O}(Ma_a^2). \quad (8)$$

In addition, it is assumed that cross-sectional variations in the mean flow quantities are negligible, leading to

$$\Psi_0(\mathbf{x}) \approx \int_0^1 \Psi_0(\mathbf{x}) dy = \langle \Psi_0 \rangle(x). \quad (9)$$

This implies a simplification, which is exact only for zero mean flow conditions. Its general validity is discussed in the next section.

Olson and Swift [40] showed that an oscillation of the material properties of the size of an acoustic perturbation only contributes to higher order terms in the resulting system of equations. Therefore, we consider material properties to be functions of the mean quantities, especially leading to

$$\beta = \frac{1}{\langle T_0 \rangle(x)}. \quad (10)$$

As we restrict ourselves to linear acoustics, the acoustic Mach number Ma_a is considered to be small. The same holds for the geometric ratio of dimensions ε in the investigated problem. Moreover, we follow the arguments of in't panhuis [16] stating that the ratio of dimensions is of the same order as Ma_a . Expressing the latter in terms of $\eta = \varepsilon^2/Ma_a^2$ in the set of equations (7f) yields individual systems of equations in different orders in ε . All expressions have to be satisfied separately by the flow. Zeroth-order terms describe the mean flow conditions inside the pore. Terms of first order determine the acoustics inside the considered field. As the acoustic Mach number can be chosen arbitrarily, we define η to be unity.

3.3. Mean transport quantities

The acoustic system of equations contains terms scaling with mean quantities. Thus, finding an analytical description for this field implies that the mean parameters can be formulated in terms of functions in $\mathbf{x}$. These formulations are derived from the mean transport equations, which are similar to equations (7f) for vanishing ε . In addition, the neglect of transversal changes in mean flow quantities is discussed.

A brief look at the transversal momentum Eq. (7d) leads directly to a block profile of the mean pressure and therefore

$$p_0(\mathbf{x}) \equiv p_0(x). \quad (11)$$

As this publication is restricted to the laminar regime, the mean flow is of Poiseuille type. According to the parallel plate solution for this flow type, the transversal velocity $v_0(\mathbf{x})$ component vanishes

$$v_0(\mathbf{x}) = 0. \quad (12)$$

Further, all transversal dependency is neglected in order to maintain compactness of the desired equations. Hence, all other mean flow quantities are treated to be constant in y , i.e. $\Psi_0(\mathbf{x}) = \Psi_0(x)$. It has to be stated that this approach is nonessential for T_0 , u_0 and thus ϱ_0 . The consequences of dropping this simplification are discussed in Section 7. This constant mean assumption may seem unphysical for the description of the mean flow conditions; however, as we focus on the acoustics inside the tube the interactions of the acoustic and mean flow profiles have to be considered. In the implicit transport equations (see [14]) some terms depend on the ratio of geometric length to acoustic boundary layer thicknesses. The viscous dissipation scales with $H_\nu(y)$ and the thermal relaxation is a function of $H_K(y)$ which read as

$$H_\nu = 1 - \frac{\cosh((1+i)Wo y)}{\cosh((1+i)Wo)} \quad \text{and} \quad H_K = 1 - \frac{\cosh((1+i)Lay)}{\cosh((1+i)La)}. \quad (13)$$

Further, the interaction of both functions, i.e. $H_\nu(x) - PrH_K(x)$, affects the TA effect. Considering their profiles with respect to the assumed mean flow profile allows a qualitative insight into the interplay of combined terms. The thinner colored lines in Fig. 3 give a qualitative impression of their profile for three different $Wo = 0.1, 1, 10$ in air. The thicker black lines indicate the two parabolic and constant dimensionless mean flow profiles. Due to the friction forces at walls all velocity components are small near the wall. For low Wo , the profile of all terms is nearly parabolic, whereas for higher values, the terms show a small peak near the wall before they almost reach a constant value far away from the wall. The intersection of the mean velocity profiles at $y = 2/3$ indicates the point from which the contribution of the acoustic profile in interaction with a constant mean flow is overestimated with increasing y .

In the acoustic equations, the product of some mean flow quantities $\psi_0(\mathbf{x})\phi_0(\mathbf{x})$ appears. Considering the product of the cross-sectional averages $\langle \psi_0(\mathbf{x}) \rangle \langle \phi_0(\mathbf{x}) \rangle$ as mentioned in the last section instead leads to a certain amount of error. Writing

$$\psi_0(\mathbf{x}) = \langle \psi_0 \rangle(x)(1 + \Psi_0(y)) \quad (14)$$

$$\phi_0(\mathbf{x}) = \langle \phi_0 \rangle(x)(1 + \Phi_0(y)) \quad (15)$$

generates four terms for the upper product

$$\psi_0(\mathbf{x})\phi_0(\mathbf{x}) = \langle \psi_0 \rangle(x)\langle \phi_0 \rangle(x)(1 + \Psi_0(y) + \Phi_0(y) + \Psi_0(y)\Phi_0(y)). \quad (16)$$

If ψ_0 denotes the mean flow velocity its transversal contribution is limited by $-2/3 \leq \langle \Psi_0(y) \rangle \leq 1/3$. In a first approximation, the ratio of the hydraulic radius of the pore to the thermal penetration depth

$$\delta_T = \frac{K}{\varrho_{\text{ref}} c_{p,\text{ref}} c_{\text{ref}} Ma} \quad (17)$$

scales the cross-sectional temperature profile. Ma denotes the reference mean flow Mach number $Ma = u_{0,u}/c_{\text{ref}}$. In the conditions met in TA applications, this ratio is very small. Thus, in contrast to neglecting the second right-hand side term of Eq. (16) assuming the other terms to be small induces only small errors. The same argument accounts for terms including ϱ_0 . Applying the approach stated in Eq. (15) on the ideal gas law (7a) directly yields the same order of magnitude for the contribution of the mean density variation in the transversal direction. Summarizing these two considerations, the error committed by the constant mean flow assumption has to be thoroughly investigated and, if necessary, the equations should be expanded to accommodate the parabolic profile.

If the impact of the neglected terms is small, the mean parameter field may be described in terms of cross-sectionally averaged quantities. At first, the mean temperature field is considered. If the transversal mean temperature dependency is neglected, the conductivity also has to be expressed in terms of a cross-sectionally averaged quantity, which describes the combined fluid and solid heat conduction. Inserting the blockage ratio $\Phi = A_{\text{gas}}/A$, this mean conductivity $\langle K \rangle$ becomes

$$\langle K \rangle = (1 - \Phi)K_S + \Phi K. \quad (18)$$

Using this quantity and introducing the Péclet number

$$Pe = \frac{c_{p,\text{ref}} \varrho_{\text{ref}} c_{\text{ref}} L}{K_{\text{ref}}} \quad (19)$$

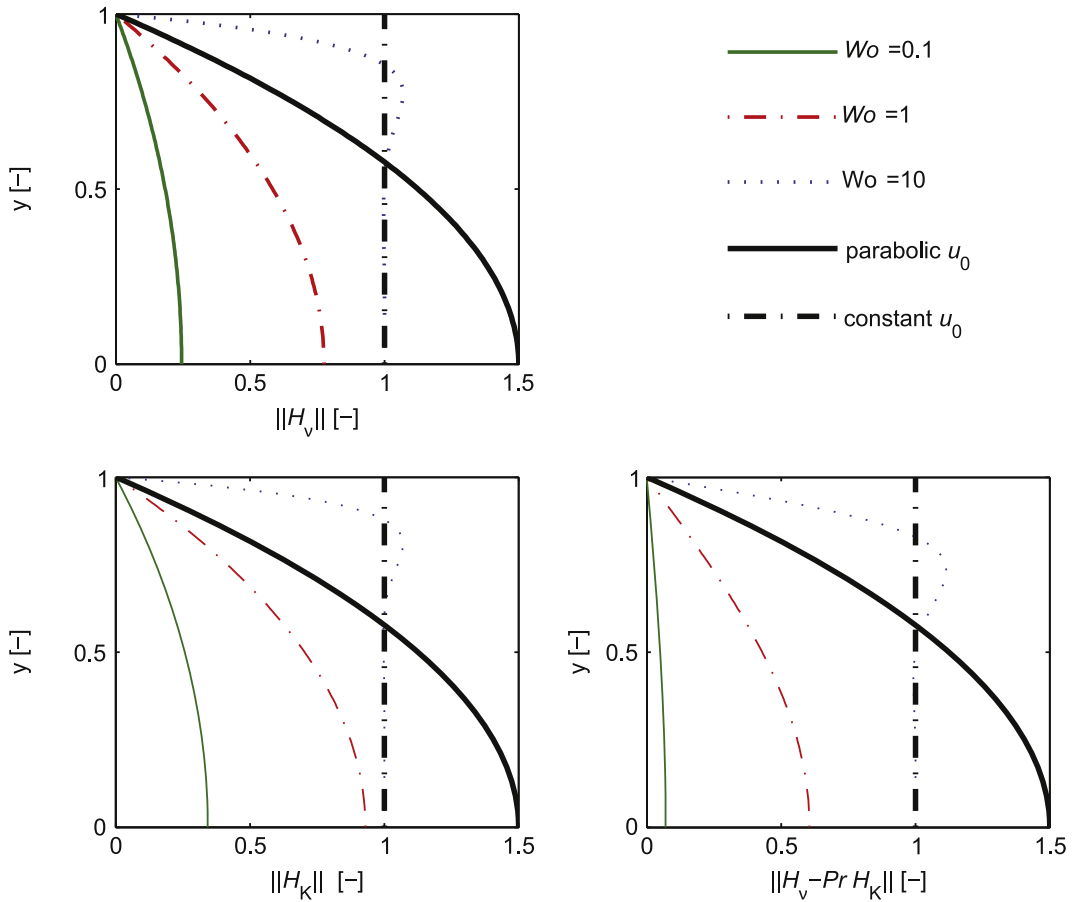


Fig. 3. The three spatial contributions to the acoustic parameters at quiescent conditions according to Eq. (13) for air at $Wo = 0.1$ (thin solid line), $Wo = 1$ (thin dash-dotted line) and $Wo = 10$ (thin dotted line). A constant (thick dash-dotted line) and a parabolic (thick solid line) mean flow profile are included for comparison. For small Wo the spacial contribution resembles a parabolic shape while at larger Wo they show a constant value at some distance to the wall.

for describing the enthalpy flow directly yields

$$c_p Pe \langle \varrho_0 u_0 \rangle \frac{d\langle T_0 \rangle}{dx} = \frac{d}{dx} \left(\langle K \rangle \frac{d\langle T_0 \rangle}{dx} \right) \quad (20)$$

with given boundary values $\langle T \rangle|_0 = 1/(\gamma - 1)$ and $\langle T \rangle|_1 = T_a/[(\gamma - 1)T_u]$.

The integral form of the averaged continuity Eq. (7b) reads

$$\langle u_0 \varrho_0 \rangle(x) = \langle u_0 \rangle(0) \langle \varrho_0 \rangle(0) = Ma \quad (21)$$

if block profile inflow conditions are met. Inserting Eq. (21) into (20) finally yields

$$c_p Pe Ma \frac{d\langle T_0 \rangle}{dx} = \frac{d}{dx} \left(\langle K \rangle \frac{d\langle T_0 \rangle}{dx} \right). \quad (22)$$

For isothermal properties, the temperature profile follows an analytic exponential solution. In general, the variation of $\langle K \rangle$ with temperature of real materials is non-negligible and is accounted for in the later computation. These investigations showed that the basic nature of the profiles remain quasi-exponential for different ceramics and metals.

Writing the separate cross-sectional average instead of the left-hand side of Eq. (21) is only exact, if one of the mean quantities does not vary in the y -direction. Due to the consideration of transversal averages, the second and third terms of Eq. (16) vanish. As discussed above, the cross-sectional variation of the mean density is small compared to its average value. Hence, neglecting the product of the y -variations in velocity and density leads to a small underestimation if we assume

$$\langle \varrho_0 u_0 \rangle(x) = Ma \approx \langle \varrho_0 \rangle \langle u_0 \rangle(x). \quad (23)$$

Finally, for closure, the axial pressure profile has to be considered. For this purpose, the mean pressure gradient term occurring in Eq. (7c) is simplified by considering the Poiseuille flow assumption. Written in a non-dimensional form and

assuming a parabolic mean velocity profile leads to

$$\frac{\partial p_0(x)}{\partial x} = \frac{\kappa}{Wo^2} \frac{\partial^2 u_0}{\partial y^2} \sim \frac{1}{Re_c \varepsilon} \langle u_0 \rangle \quad (24)$$

Further, if the Reynolds number

$$Re_c = \frac{Rc}{\nu} \quad (25)$$

is larger than the inverse of the ratio of dimensions ε , which is often valid for TA applications, the right-hand side term of Eq. (24) becomes small. Under those conditions, the influence of the axial mean pressure gradient vanishes. Thus, it is assumed that the axial mean pressure profile is constant with respect to its mean quantity. This leads to a pure mean temperature dependency of the axial $\langle u_0 \rangle$ profile, which is order of magnitudes larger than the changes caused by the mean pressure gradient. The impact of the pressure gradient, which physically causes the mean flow, sets the inlet velocity. Using this assumption, all mean quantities may be computed. Further, if all the presented assumptions apply, those terms in the acoustic system of equations that have contributions in terms of v_0 , the y -derivative of mean quantities, or the axial derivative of the mean pressure, may be neglected.

3.4. Linearized Navier–Stokes equations of a slab pore

The analytical formulations derived in the last section sufficiently describe the mean flow terms occurring in the acoustic set of equations. The acoustic field is governed by the terms of first order in ε and in this context they are denoted as the linearized Navier–Stokes equations (LNSE) of a slab pore. In the most general form they read as

$$q_1 = \frac{\gamma}{\gamma - 1} p_1 - \frac{\rho}{T} T_1 \quad (26a)$$

$$\left(i\kappa + \frac{\partial u}{\partial x} \right) q_1 + u \frac{\partial q_1}{\partial x} + \frac{\partial q_1}{\partial x} u_1 = -q \left(\frac{\partial v_1}{\partial y} + \frac{\partial u_1}{\partial x} \right) \quad (26b)$$

$$q \left(i\kappa + \frac{\partial u}{\partial x} \right) u_1 + \underbrace{u q \frac{\partial u_1}{\partial x}}_{\Lambda_1} = -\frac{\partial p_1}{\partial x} + \frac{\kappa \mu}{Wo^2} \frac{\partial^2 u_1}{\partial y^2} \quad (26c)$$

$$0 = -\frac{\partial p_1}{\partial y} \quad (26d)$$

$$i\kappa c_p q T_1 + \underbrace{u q c_p \frac{\partial T_1}{\partial x}}_{\Lambda_2} + c_p \frac{\partial T}{\partial x} u_1 + c_p \frac{\partial T}{\partial x} u q_1 = i\kappa p_1 + T u \frac{\partial p_1}{\partial x} + \frac{\kappa K}{2La^2} \frac{\partial^2 T_1}{\partial y^2} \quad (26e)$$

$$i\kappa c_s q_s T_{s,1} = \frac{\kappa K_s}{2La_s^2} \frac{\partial^2 T_{s,1}}{\partial y^2}, \quad (26f)$$

where all first-order quantities are defined in two-dimensional space, i.e. $\Psi_1 = \Psi_1(x, y)$. Here onward the index 0 as well as the spatial averaging $\langle \rangle$ symbol are dropped for all mean quantities. It is important to note that the gray boxed terms explicitly include mean velocity contributions. This is the main difference to previous publications [14,16,29,30]. All these terms describe the explicit, spatially averaged impact of mean flow inside a pore. The two-dimensional acoustic quantities are affected in- and outside of the thermo-viscous acoustic boundary layers. The system of equations of in't panhuis et al. [16] is recovered, if all terms including mean flow effects are dropped.

4. Transversally averaged acoustic perturbation equations

The cross-sectional averaging of the LNSEs of a pore (Eqs. (26)) leads to a system of one-dimensional, coupled differential equations. This set can then be evaluated numerically with little effort. Nevertheless, the averaged equations contain numerous terms. To avoid human errors in this derivation step, the process was carried out with a computational software capable of symbolic equation handling [42]. The derivation of the intermediate and final formulations in written form is very extensive. The reader is referred to Table 2, containing an overview of each single step of the derivation procedure. The integration of the trivial transversal momentum equation (26d) in step 1 yields a constant pressure profile over the pore width. In the second step, applying the method of Green's functions on the axial momentum equation (26c) and modeling the term Λ_1 leads to an analytical description of the transversal profile of the acoustic velocity u_1 . With these results, the solution for the energy equations ((26e) and (26f)) is obtained in a similar manner (step 3). The fluctuating temperature in the solid domain $T_{1,s}$ is used to express the non-constant coupling wall temperature T_1 in the fluid domain. Inserting the expressions of these steps in terms of acoustic velocity u_1 , pressure p_1 and temperature T_1 into the gas law (26a) (step 4)

Table 2Overview of the segregated mathematical steps applied to the LNSEs that lead to a transport ODE system in terms of p_1 and u_1

Step	Equations	Applied methods	Result
1	(26d)	Transversal integration	$p_1(x, y) = p_1(x)$
2	(26c)	(a) Modeling of Λ_1 (b) Green's function method (c) Transversal integration	$\frac{\partial p_1}{\partial x} = A_{11}(x)\langle u_1 \rangle(x)$
3	(26e) and (26f)	(a) Modeling of Λ_2 (b) Green's function method (c) Boundary coupling (d) Transversal integration	$\langle T_1 \rangle(x) = f(\langle u_1 \rangle(x), \langle p_1 \rangle(x))$
4	(26a)	(a) Transversal integration (b) Inserting	$\langle \varrho_1 \rangle(x) = f(\langle u_1 \rangle(x), \langle p_1 \rangle(x))$
5	(26b)	(a) Transversal integration (b) Inserting	$\langle v_1 \rangle(x) = 0 \quad \frac{\partial u_1}{\partial x} = A_{21}(x)\langle u_1 \rangle(x) + A_{12}(x)\langle p_1 \rangle(x)$
6		Combination of Steps 2 and 5	$\frac{d}{dx} \begin{pmatrix} p_1 \\ \langle u_1 \rangle \end{pmatrix} = \mathbf{A}(x) \begin{pmatrix} p_1 \\ \langle u_1 \rangle \end{pmatrix}$

yields an expression for the density fluctuations ϱ_1 . Eliminating all terms except for u_1 , p_1 , finally leads to a second ordinary differential equation (ODE) of the transversal quantities (step 5) which forms a system of transport equations with the solution of step two (step 6).

A standard analytical method for solving ODEs with a not vanishing source term is Green's functions method. Applying it to inhomogeneous differential equations allows one to find an analytical solution for any right-hand side inhomogeneity term. The present derivation is based on that technique. Consequently, the general procedure is described first.

The LNSEs are a set of coupled differential equations containing partial derivatives in all coordinate directions. For the purpose of finding an analytical solution in the transversal (y) direction, the relevant acoustic parameters are split into a product of $\Psi_1(\mathbf{x}) = \psi_1(x)\vartheta(y)$, applying the method of separation of variables [43]. The resulting differential operator $\mathcal{L}$ in the transversal direction can be identified and applied to the corresponding basic equation [44]

$$\mathcal{L}[\vartheta] = b(y) \quad (27)$$

and its Dirichlet boundary condition (BC)

$$\vartheta(y_m) = g_m. \quad (28)$$

The corresponding GF $G(y, \hat{y})$ describes the solution for a differential equation with the same differential operator $\mathcal{L}$ in y and ErsatzBC the same boundary condition, but a special right-hand side inhomogeneity. This inhomogeneity is a Dirac impulse in $\hat{y}$. The Dirac function is the neutral element of the convolution operation, thus the resulting solution of this problem can be used to find solutions for any other inhomogeneity. The basic form of the GF for a differential operator $\mathcal{L}_j$ of the form

$$\mathcal{L}_j \vartheta = \left[1 - \frac{1}{\alpha_j^2} \frac{\partial^2}{\partial y^2} \right] \vartheta. \quad (29)$$

reads

$$G(y|\hat{y}) = -\alpha H(y - \hat{y}) \sinh(\alpha(y - \hat{y})) + C_1(\hat{y})e^{\alpha y} + C_2(\hat{y})e^{-\alpha y}, \quad (30)$$

with $C_{1,2}$ depending on the boundary conditions g_m . The index j denotes the equation the approach is applied to; in this case either ν or K . Convoluting $G(y|\hat{y})$ with the right-hand side of Eq. (27) in $\hat{y}$ and taking the BC into account yield

$$\vartheta(y) = G(y, \hat{y}) * b(\hat{y}) + \frac{1}{\alpha_j^2} \sum_m g_m \frac{\partial G}{\partial \hat{y}} \Big|_{\hat{y}=y_m}. \quad (31)$$

where the index m indicates a boundary. This solution is valid as long as α_j is independent of y . This quantity may still vary along any other direction.

4.1. Closure assumptions

We restrict ourselves to the goal of finding an analytical solution for the problem considered. Due to the different nature of Eqs. (26b)–(26f) a coupled solution of the equations does not exist. However, solving the weakly coupled equations separately in segregated steps yields an insight as to which parts impede from finding an analytical solution. The definition of closure assumptions for such terms gives access to an analytical approach of the equations. Before the cross-sectional averaging procedure is carried out, the applied modeling is introduced.

The axial momentum equation (26c) can be rearranged such that the differential operator for the axial velocity u_1 becomes

$$\mathcal{L}_v[u_1] = \left[\varrho \left(i\kappa + \frac{\partial u}{\partial x} \right) + \underbrace{u\varrho \frac{\partial}{\partial x}}_{\Lambda_1} - \frac{\kappa\mu}{Wo^2} \frac{\partial^2}{\partial y^2} \right] u_1. \quad (32)$$

Unfortunately, due to the convective term $u\varrho \partial u_1 / \partial x$ the operator $\mathcal{L}_v[u_1]$ is not only an operator in the y direction. Thus, an analytical GF cannot be found for this operator, only numerical solutions are available. This is in conflict with the aim of this paper to provide an analytical solution for the cross-sectional part of the set of governing (26). Therefore, the term indicated by Λ_1 either has to be modeled in terms of u_1 or formulated as inhomogeneity on the right-hand side with defined transversal shape (i.e. in this step p_1). The latter approach implies that the term Λ_1 is no longer a function of the axial acoustic velocity u_1 and thus does not influence the formulation of the GF any more.

A similar convective term occurs in the fluid energy equation (26e). Substituting the acoustic density by Eq. (26a) facilitates writing the differential operator in terms of the fluctuating temperature T_1

$$\mathcal{L}_K[T_1] = \left[\left(i\kappa c_p \varrho - c_p u \frac{\partial T}{\partial x} \right) + \underbrace{u\varrho c_p \frac{\partial}{\partial x}}_{\Lambda_2} - \frac{\kappa K}{2La^2} \frac{\partial^2}{\partial y^2} \right] T_1. \quad (33)$$

This expression also includes a term (Λ_2), depending on the axial derivative of the parameter itself. Due to its similar nature, the modeling is chosen in accordance with Λ_1 . As the transversal profile of the acoustic velocity u_1 is defined by the solution of Eq. (26c), all right-hand side terms of Eq. (26e) are known, and thus can be invoked in the analytical solution. Therefore, a reasonable modeling for both convective terms Λ_1 and Λ_2 has to be found before the application of the GF method is carried out. Finding a model that represents the a priori unknown relevant physical impact of the convective term Λ_1 is not straightforward. Iterative methods are often used to solve the problem of modeling an unknown term: Starting from a relative simple approach, the achieved solution can be used as a model in a following step. This procedure is repeated until the desired accuracy is reached. The convergence of this method strongly depends on the initial approach. Here, we present two alternative derivations. If higher accuracy is demanded, the solutions achieved may be used as a starting point for an iterative derivation.

The first suggestion is a very general approach. Solving the equations without the bothersome terms Λ_i yields an analytical solution, which can be inserted in an iterative procedure. Especially the first iteration is advantageous for developing an understanding of the derivation process. Neglecting the convective terms reduces the length of the system of equations, which makes them manageable in a better way. Further, one may argue that at least for small convective scales in terms of Ma , the contribution of such components in an acoustically compact region should not affect the acoustic fields decisively.

The second way is rather problem restrictive. As an initial solution, the zero mean flow solution is used. In contrast to a neglect of Λ_i , it describes a physical problem for special mean conditions. This idea leads to an immense increase in the complexity of the system. The number of emerging terms is many times that of the first approach. As this modeling is part of the first and the second of six derivation steps leading to the final set of ODEs, this increase causes a vast rise in the number of terms in the resulting set of equations. Thus, a simplified version of inserting the quiescent mean solution can be chosen by only considering the spatially averaged contribution of the terms $\partial \psi_1 / \partial x = (\partial \langle \psi_1 \rangle / \partial x)_{u_0=0}$. This approach includes a more physical contribution of the term than completely neglecting it, like in the previous approach. However, this simplification is not motivated by physical considerations. One argument that may justify this step is the later cross-sectionally averaging of the entire system of equations, which makes it a look-ahead approach. The reader should be aware that the models presented here are intended to be used in the initial phase of an iterative derivation. Taking this averaging into account may lead to less accurate results of the first iteration step. Thus, this simplification only leads to slower convergence.

In this publication, two of these modeling assumptions are carried out in order to observe their performance thoroughly: (i) In **M I** both convective terms are neglected. (ii) For **M II** the system of Eqs. (26a)–(26f) is solved for $Ma = 0$. This solution, where all gray boxed terms are zero, is inserted in a transversally averaged form. Table 3 summarizes the terms of both model assumptions. An iterative insertion of the corresponding solutions is beyond the scope of this publication. In the following sections, the modeled terms are replaced by their representative Λ_i indicating any contributions of the differently modeled source terms, which here are considered constant in the transversal direction.

4.2. Cross-sectional averaging of the LNSEs

Due to the large number of terms that occur in the spatial averaging process, the pertinent terminology is explained. The blackboard bold capital letters (e.g. $\mathbb{E}$) indicate the equation, in which the replacement character is introduced at. The overbar ($\overline{}$) marks the terms stemming from transversally averaged equations. The indices refer to three different issues: (a) the corresponding prefactors (e.g. p_1) the term multiplies with, (b) terms that are included in the integral (e.g. F_v) or (c) the inclusion of a modeled term (Λ_i). Table 4 lists the abbreviations and indices and gives reference to the corresponding

Table 3

Overview over the modeled terms: For model **M I**, the convective terms are neglected. **M II** uses a spatially averaged, quiescent mean solution.

Term	Character	Model	
		M I	M II
$\frac{\partial u_1}{\partial x}$	Λ_1	0	$\left. \frac{\partial \langle u_1 \rangle}{\partial x} \right _{u_0=0}$
$\frac{\partial T_1}{\partial x}$	Λ_2	0	$\left. \frac{\partial \langle T_1 \rangle}{\partial x} \right _{u_0=0}$

Table 4

Replacement characters and their indexing.

Character	Eq.	Description	Index	Description
E	(26e)	Energy conservation	p_1	Acoustic pressure
G	(26a)	Gas law	p'_1	Axial derivative of p_1
$\bar{G}$	(26a)	Averaged gas law	u_1	Acoustic velocity
$\bar{k}$	(26b)	Intermediate averaged continuity	u'_1	Axial derivative of u_1
			T_1	Acoustic temperature
$\bar{k}$	(26b)	Final averaged continuity	q_1	Acoustic density
T	(26e)	Solution of energy	q'_1	Axial derivative of q_1
	(26f)	Conservations	ϵ_S	Containing heat capacity ratio
X	(26c)	Axial momentum	F_ν	Transversal viscous part
		conservation	F_K	Transversal thermal part
Λ_1	(26c)	Modeled momentum term	Λ_1	Containing modeled term 1
Λ_2	(26e)	Modeled enthalpy term	Λ_2	Containing modeled term 2

equations in [Appendix A.1](#). The averaging of the parameter u_1 , T_1 and q_1 proceeds in the same order as in the zero mean flow derivation processed by in't panhuis [15].

Like the mean transversal momentum Eq. (7d), the acoustic momentum conservation in y (26d) reveals that the acoustic pressure $p_1(x, y) = p_1(x)$ is constant over the cross-section of the channel.

Applying the assumptions **M I**, **M II** defined in [Section 4.1](#) into the momentum Eq. (26c) and using the differential operator of equation (29) allows one to rewrite it as

$$\mathcal{L}_\nu[u_1] = \mathbb{X}_{\Lambda_1} + \mathbb{X}_{p_1'} \frac{\partial p_1}{\partial x} \quad (34)$$

$$u_1|_{\pm 1} = 0 \quad (35)$$

where the coefficient α_ν^2 in the differential operator $\mathcal{L}_\nu[u_1]$ (see Eq. (31)) stands for

$$\alpha_\nu^2 = e \left(i\kappa + \frac{\partial u}{\partial x} \right) \frac{Wo^2}{\kappa\mu}. \quad (36)$$

Arnott et al. [17] solved the GF problem of a similar differential operator, a constant inhomogeneity and a value of zero at the boundaries. Introducing the function derived in that work,

$$F_j(y) = 1 - \frac{\cosh(\alpha_j y)}{\cosh(\alpha_j)} \quad (37)$$

the solution of the transversal problem is obtained

$$u_1 = \int_0^1 G(y, \hat{y}) \delta(y - \hat{y}) d\hat{y} \left(\mathbb{X}_{\Lambda_1} + \mathbb{X}_{p_1'} \frac{\partial p_1}{\partial x} \right) = F_\nu \left(\mathbb{X}_{\Lambda_1} + \mathbb{X}_{p_1'} \frac{\partial p_1}{\partial x} \right). \quad (38)$$

Finally, we now perform the averaging in the y -direction. This yields

$$\langle u_1 \rangle = \int_0^1 u_1 dy = (1 - f_\nu) \left(\mathbb{X}_{\Lambda_1} + \mathbb{X}_{p_1'} \frac{\partial p_1}{\partial x} \right). \quad (39)$$

where the lowercase f denotes the Rott [12] function

$$f_j = 1 - \langle F_j \rangle = \frac{\tanh(\alpha_j)}{\alpha_j}. \quad (40)$$

Eq. (39) is the first acoustic transport equation, which will be solved numerically later on.

Next, the density fluctuation ϱ_1 is replaced in the energy equation (26e). Transforming it along with the solid energy equation (26f) into the same form as Eq. (34) and setting the coefficients that are constant along the transverse direction in the corresponding differential operator to

$$\alpha_K^2 = \left[\varrho c_p \left(-i\kappa + \frac{u}{T} \frac{\partial T}{\partial x} \right) \right] \frac{2La^2}{\kappa K} \quad (41)$$

$$\alpha_S^2 = \frac{ic_S \varrho_S}{\phi^2} \frac{2La_S^2}{K_S} \quad (42)$$

allows one to write them in combination with their coupling conditions as

$$\mathcal{L}_K[T_1] = \mathbb{E}_{\Lambda_2} + \mathbb{E}_{\Lambda_1, F_\nu} F_\nu + (\mathbb{E}_{p_1} + \mathbb{E}_{p_1, F_\nu} F_\nu) p_1 + (\mathbb{E}_{p_1'} + \mathbb{E}_{p_1', F_\nu} F_\nu) \frac{\partial p_1}{\partial x} \quad (43)$$

$$\mathcal{L}_S[T_{1,S}] = 0 \quad (44)$$

$$T_1|_{\pm 1} = T_{1,S}|_{\pm 1} = T_b \quad (45)$$

$$K \frac{\partial T_1}{\partial y} \Big|_{\pm 1} = K_S \frac{\partial T_{1,S}}{\partial y} \Big|_{\pm 1}. \quad (46)$$

The solution of the differential equations with symmetry conditions at $y=0$ and $y=1+R_S/R$ leads to formulations of the temperature oscillations T_1 , $T_{1,S}$ that contain the coupling wall boundary temperature T_b

$$T_1 = \mathbb{E}_{\Lambda_2} F_K + \mathbb{E}_{\Lambda_1, F_\nu} F_{K,\nu} + (\mathbb{E}_{p_1} F_K + \mathbb{E}_{p_1, F_\nu} F_{K,\nu}) p_1 + (\mathbb{E}_{p_1'} F_K + \mathbb{E}_{p_1', F_\nu} F_{K,\nu}) \frac{\partial p_1}{\partial x} + (1 - F_K) T_b \quad (47)$$

$$T_{1,S} = (1 - F_K) T_b. \quad (48)$$

Inserting these equations into Eqs. (45) and (46) allows one to replace $T_{1,S}$ and T_b in the later formulation of the acoustic temperature transport. The constant terms in $\mathcal{L}_K[T_1]$ and $\mathcal{L}_S[T_{1,S}]$ reveal again thermal Arnott functions, which only differ in the factor α_j . The mixed term

$$F_{K,\nu} = \frac{-F_K + \mathfrak{Pr} F_\nu}{-1 + \mathfrak{Pr}} \quad (49)$$

in Eq. (47) is the result of the convolution of the GF and the viscous Arnott function. In this formulation, the square ratio of the operator constants α_K^2/α_ν^2 was replaced by $\mathfrak{Pr}$ with respect to the implicit solution **IMP**. When mean flow is neglected, this ratio reduces to the Prandtl number Pr of the fluid. Next, the Neumann condition (46) can be applied to find

$$\begin{aligned} \frac{T_b}{\mathbb{E}_{\epsilon_S}} &= \mathfrak{Pr} \mathbb{E}_{\Lambda_2} + \mathbb{E}_{\Lambda_1, F_\nu} \left(\frac{f_{K,\nu}}{f_K} - 1 \right) \\ &+ \left[\mathfrak{Pr} \mathbb{E}_{p_1} + \mathbb{E}_{p_1, F_\nu} \left(\frac{f_{K,\nu}}{f_K} - 1 \right) \right] p_1 \\ &+ \left[\mathfrak{Pr} \mathbb{E}_{p_1'} + \mathbb{E}_{p_1', F_\nu} \left(\frac{f_{K,\nu}}{f_K} - 1 \right) \right] \frac{\partial p_1}{\partial x}, \end{aligned} \quad (50)$$

whereas $\mathbb{E}_{\epsilon_S}$ and ϵ_S are substitutes for

$$\mathbb{E}_{\epsilon_S} = \frac{\epsilon_S}{\mathfrak{Pr}(1 + \epsilon_S)} \quad (51)$$

$$\epsilon_S = \frac{K \mathfrak{Pr} f_K}{K_S Pr f_S}. \quad (52)$$

Inserting Eq. (50) into Eq. (47) and introducing new variables $\mathbb{T}$, $\mathbb{E}$ (see Appendix A.1) finally yields the expression for the temperature fluctuation

$$T_1 = \mathbb{T}_{\Lambda_1} + \mathbb{T}_{\Lambda_2 F_K} F_K + \mathbb{E}_{\Lambda_1, F_\nu} F_{K,\nu} + (\mathbb{T}_{p_1} + \mathbb{T}_{p_1, F_K} F_K + \mathbb{E}_{p_1, F_\nu} F_{K,\nu}) p_1 + (\mathbb{T}_{p_1'} + \mathbb{T}_{p_1', F_K} F_K + \mathbb{E}_{p_1', F_\nu} F_{K,\nu}) \frac{\partial p_1}{\partial x}. \quad (53)$$

This solution as well as its cross-sectional averaged form is now merged with the acoustic ideal gas law (26a) to obtain a formulation for the density fluctuation ϱ_1 and its transversal average $\langle \varrho_1 \rangle$

$$\varrho_1 = (\mathbb{T}_{\Lambda_1} + \mathbb{T}_{\Lambda_2 F_K} F_K + \mathbb{E}_{\Lambda_1, F_\nu} F_{K,\nu}) \mathbb{G}_{T_1} + [\mathbb{G}_{p_1} + (\mathbb{T}_{p_1} + \mathbb{T}_{p_1, F_K} F_K + \mathbb{E}_{p_1, F_\nu} F_{K,\nu}) \mathbb{G}_{T_1}] p_1 + (\mathbb{T}_{p_1'} + \mathbb{T}_{p_1', F_K} F_K + \mathbb{E}_{p_1', F_\nu} F_{K,\nu}) \mathbb{G}_{T_1} \frac{\partial p_1}{\partial x} \quad (54)$$

$$\langle \varrho_1 \rangle = \overline{\mathbb{G}}_{\Lambda_1, \Lambda_2} + \overline{\mathbb{G}}_{p_1} p_1 + \overline{\mathbb{G}}_{p_1'} \frac{\partial p_1}{\partial x}. \quad (55)$$

The average of the transversal acoustic component $\langle v_1 \rangle$ vanishes in the integral form of the continuity (26b). Inserting the formulation for $\langle \rho_1 \rangle$ and $\partial \langle \rho_1 \rangle / \partial x$ into the cross-sectional averaged conservation of mass (Eq. (26b)) yields

$$\frac{\partial \langle u_1 \rangle}{\partial x} = \overline{\mathbb{K}}_{\Lambda_1, \Lambda_2} + \overline{\mathbb{K}}_{p_1} p_1 + \overline{\mathbb{K}}_{p_1'} \frac{\partial p_1}{\partial x} + \overline{\mathbb{K}}_{u_1'} \langle u_1 \rangle. \quad (56)$$

Finally, using Eq. (39) leads to the second transport ODE

$$\frac{\partial \langle u_1 \rangle}{\partial x} = \overline{\mathbb{K}}_{\Lambda_1, \Lambda_2} - \frac{\overline{\mathbb{K}}_{p_1'} \mathbb{X}_{\Lambda_1}}{\mathbb{X}_{p_1'}} + \overline{\mathbb{K}}_{p_1} p_1 + \left[\overline{\mathbb{K}}_{u_1'} + \frac{\overline{\mathbb{K}}_{p_1'}}{(1-f_v) \mathbb{X}_{p_1'}} \right] \langle u_1 \rangle \quad (57)$$

that describes the axial transport of the acoustic velocity $\langle u_1 \rangle$ solely in terms of pressure fluctuation. The first two terms as well as the term $\mathbb{X}_{\Lambda_1}$ in the first transport ODE (Eq. (39)) show the impact of the closure terms. For **M I** the terms $\mathbb{X}_{\Lambda_1}$ and $\overline{\mathbb{K}}_{\Lambda_1, \Lambda_2}$ vanish. If the closure assumptions $\Lambda_{1,2}$ are formulated in terms of u_1 and p_1 (e.g. **M II**) these terms contribute to all components of the system matrix **A** of the transport equations (39) and (57) written as

$$\frac{d}{dx} \begin{pmatrix} p_1 \\ \langle u_1 \rangle \end{pmatrix} = \mathbf{A}(x) \begin{pmatrix} p_1 \\ \langle u_1 \rangle \end{pmatrix}. \quad (58)$$

As only cross-sectional averaged acoustic quantities or quantities that do not depend on the transversal direction are considered from now on, the angular brackets are omitted.

5. Spatial propagation and scattering matrices

The accuracy of the closure assumptions chosen is validated against an existing one-dimensional solution denoted by **IMP** and a full CFD computation of one single pore. At first the x distributions for a single frequency are compared. Then, the frequency dependent scattering behavior is investigated in terms of scattering matrices.

5.1. Scattering matrices

In the investigated problem, the spatial distribution of $p_1(x)$ and $u_1(x)$ depends on the investigated frequency. Thus, a full characterization of the quality of closure terms by comparison of the spatial results for one frequency is impossible. Furthermore, the boundary conditions, i.e. the phase between the acoustic variables and their amplitude ratio strongly affect the solution. Hence, only one spatial profile is not enough to determine the full impact of mean flow. Therefore, a more general investigation tool has to be chosen: the acoustic scattering matrices.

Standard low-order network models of wave propagation [45–47] based on one-dimensional acoustics use scattering matrices for the stability prediction of acoustic systems. Guedra et al. [48] used measured TA core data of this type to predict the onset of various TA engines. Scattering matrices describe the linear correlation of the characteristic wave amplitudes, which are often denoted as incoming and outgoing waves, in a causal sense. These complex valued amplitudes correlate to the acoustic variables by

$$p_1 = \rho c (\mathcal{F} + \mathcal{G}) \quad (59)$$

$$u_1 = \mathcal{F} - \mathcal{G}. \quad (60)$$

Here, the wave traveling in the positive axial direction is named $\mathcal{F}$ whereas the upstream traveling wave is designated $\mathcal{G}$.

A variety of numerical and experimental identification methods of such matrices either apply the two-load-one-source location (see [49]) or the two-source location method introduced by Munjal et al. [50]. The common idea of both approaches is to create a set of two independent acoustic states. Inserting these into the definition of the scattering matrix

$$\begin{pmatrix} \mathcal{F}_u \\ \mathcal{G}_d \end{pmatrix} = \begin{bmatrix} t_{ud} & r_{dd} \\ r_{uu} & t_{du} \end{bmatrix} \begin{pmatrix} \mathcal{F}_d \\ \mathcal{G}_u \end{pmatrix} \quad (61)$$

allows one to determine the linear coefficients for an acoustic element. Here, the first method is applied to the system by shifting the phase of the pressure boundary condition by $\pi/2$. The integration of the ODEs is processed for both sets of BCs and a discrete number of frequencies. The same integration technique as for the investigation of the spatial distribution of p_1 and u_1 is applied in this case.

5.2. Numerical integration of the thermoacoustic transport equations

The frequency dependent boundary value problem is now solved numerically in the complex space using standard integration methods. Given the boundary conditions at the cold reference end, a standard numerical integration method can be used to solve the set of equations for the acoustic field (58) for a predefined mean temperature $T(x)$ and velocity distribution $u(x)$.

Here, a standard mathematics software tool is used to compute the relevant spatial distributions in segregated steps. At first, a shooting method [51] with an accuracy of 10^{-12} is used to solve Eq. (20) in combination with given mean

temperatures at both ends and a given cold side inlet Ma number. In a second step, quadratic interpolated values of the resolved temperature distribution $T(x)$ are used to solve the ODE system for the acoustic field. This approximation is applicable, because the highest order spatial derivative of T occurring is $\partial^2 T / \partial x^2$. An explicit linear multistep Adams–Bashforth method [51] of variable order and step size is applied. For quiescent conditions, the implementation was tested against the existing freeware DeltaEC [24,52] from the Los Alamos group around G.W. Swift. The code achieved deviations below 1 percent, which we ascribe to the different discretization and implementation of material properties.

5.3. Validation cases

Both closure approaches are compared for up to three different cases. One condition without mean flow $Ma_0 = 0$ and two conditions with mean flow at $Ma \approx 3 \times 10^{-4}$ are investigated. For the latter two different Péclet numbers in a pore of $\varepsilon = 1/52$ are considered. First, a linear mean temperature profile is caused by a vanishing Pe number, which is denoted by Ma_{lin} . Then, convection is increased by setting $Pe \approx 30$. This causes a clearly exponential-like distribution (Ma_{exp}). Table 5 summarizes the validation cases used.

The presented system of thermoacoustic transport equations is validated against the existing implicit solution (IMP) of in't panhuis [37] and Swift [14] and full compressible CFD finite volume simulations. The same solution method is applied to the implicit and explicit one-dimensional ODEs for better comparability. The thermo-physical properties of the fluid are interpolated from tabulated values [53].

At first, the spatial evolution at $Wo = 1.74$ is observed for Ma_0 and Ma_{exp} using air at standard conditions. The changes in convective contribution to the mean heat flux from case Ma_{lin} to Ma_{exp} allow inferences on the importance of the temperature distribution in combination with mean flow.

To compare the results to a full CFD simulation, the 1D computation is adapted in two points. The boundary values for p_1 and u_1 are extracted from the periodic CFD result at $x=0$ and applied to the 1D simulations. Further, the solid temperature is fixed, because the reference simulation cannot capture conjugate heat transfer. As a direct consequence of this restriction, the factor ε_S is set to zero in the one-dimensional linear PDEs.

The scattering matrices are computed for a Womersley number range of 0.02–5. The frequencies ω are determined such that they form an equally dense grid in the Wo space. For each frequency the system of equations (58) is solved once. From these data points the complex valued scattering matrix is constructed.

5.3.1. Full CFD simulation

The validation simulations are performed with the open source CFD code OpenFOAM [54]. The full laminar compressible Navier–Stokes equations are solved for the domain sketched in Fig. 2. The geometrical scales are a length of $L = 0.052$ m and a half pore height of $R = 1 \times 10^{-3}$ m. The wall temperatures are fixed in time. The simulation domain of the pore is extended by two free field areas with a length of 0.05 m, which are appended at both ends of the stack channel. The block structured grid consists of cells with a maximum axial size of $\Delta x = 0.1$ mm. Outside the pore, the grid is coarsened using a stretching factor of maximum 1.15 avoiding acoustic reflections at cell interfaces. The transversal direction is discretized by 10 cells. They experience an exponential refinement towards the solid wall boundary. Applying discretizing schemes that are of at least fourth order, this grid reproduces the smallest acoustic boundary layers considered here. Compared to the analytical solution [14] an overall accuracy of at least 99 percent was reached. Due to the fine meshing the acoustic Courant–Friedrich–Lewy (CFL) condition requires to use a time step of $\Delta t = 1.25 \times 10^{-7}$ s.

The mean temperature at the wall is fixed such that the cross-sectionally averaged downstream temperature deviates up to 1 percent between the cases. The acoustic signals are excited at the upstream and downstream BCs located at both ends of the free fields. Those non-reflecting boundary conditions of the Euler characteristic BC [55] type use local one-dimensional inviscid (LODI) [56] relations. They are enhanced with a relaxation term that was proposed by Rudy and Strikwerda [57] and a plane wave masking filter technique [58]. This combination showed a good non-reflecting behavior in different CFD codes [59,60]. A time dependent term is further added for exciting the acoustic wave that is entering the domain. The amplitude is fixed such that the acoustic velocity u_1 reaches 5 percent of the characteristic velocity p/qc .

The single frequency simulation needs to reach a quasi-steady state. Thus, it is processed until the acoustic variables p_1 , u_1 , T_1 at three predefined plane locations (upstream end, downstream end and center of the pore) do not vary for more than 0.1% from the last three periods of oscillation. The time line is extracted with a sufficient temporal resolution ($\Delta t = 200/T$). Averaging over five oscillation periods is performed for post-processing.

Table 5
Overview over the validation cases.

Case	Designation	Mach number	Péclet number
Case 1	Ma_0	$Ma = 0$	$Pe = 0$
Case 2	Ma_{lin}	$Ma \approx 3 \times 10^{-4}$	$Pe \rightarrow 0$
Case 3	Ma_{exp}	$Ma \approx 3 \times 10^{-4}$	$Pe \approx 30$

Before the spatial averaging at the observation frequency is applied, the time domain information is transformed into frequency space and filtered.

The multi-frequency simulations are performed applying pre-filtered random binary excitation signals. The CFD/SI method described in [61] is used for post-processing. This method and its successor the LES/SI method (see [36,62,63]) apply the Wiener-filter method on SISO (single input single output) and MISO (multiple input single output) systems to identify the frequency dependent transfer coefficients from time varying signals and responses. A laminar regime is observed, hence, no turbulent noise is induced into the simulation. The accuracy of such simulations scales with the signal length and the inverse of the frequency. The errors of results for wavelengths, fitting at least half inside the observed time line, are approximated to be in the range of ± 1 percent.

6. Results

6.1. Spatial propagation for a single frequency

The solid lines depicted in Fig. 4 represent the axial profiles of amplitude of the acoustic pressure (a) and the velocity fluctuation (b). Two cases are investigated: quiescent conditions (Ma_0) and the mean flow affected case $Ma_{exp} \approx 3 \times 10^{-4}$. The displayed phase Φ (dashed with empty symbols) is displayed relative to the reference phase of the acoustic pressure p_1 located at the upstream end of the pore.

All boundary conditions are chosen to be equal except for BC of the mean upstream velocity. Nevertheless, the upstream pressures ($p_1|_{x=0}$) change with Mach number because of the different TA interaction processes and the imperfect reflectivity of the BCs. The variation of the axial evolution of $|p_1|$ for approximately 50 percent visualizes the influence of TA effects on the pressure field. Additionally, higher mean flow causes an increase in phase change of p_1 over the pore. The velocity distribution remains almost unaffected. This may be explained by the imposed characteristic waves and their phase difference of $\Phi(p_1) - \Phi(u_1) \approx \pi/2$ creating a standing wave-like condition near a velocity anti-node.

Fig. 5 shows the relative magnitude and absolute phase error of the three investigated models (**IMP**, **M I**, **M II**) compared to the CFD solution for quiescent mean flow ($Ma_0 = 0$). The perfectly matching error curves indicate an identical behavior of all three tested quasi-one-dimensional models in comparison to the CFD simulation for the zero mean velocity condition. The maximum error in amplitude never exceeds 2.5 percent. The combination of this magnitude value and phase errors of up to 3° reveals a very good agreement of both simulation types. The differences are mainly ascribed to the different, multi-dimensional CFD conditions at the pore in- and outlet. The pressure amplitude and phase error exceeds the velocity deviation by a factor of two.

In the presence of mean flow, the 1D models do not behave identically (see Fig. 6). The 1D acoustic pressure alternates around the CFD data. The decrease of the $|\Delta u_1|$ near the downstream position indicates a similar behavior at a larger length scale. The phase errors of all 1D models behave equal: $\Phi(\Delta u_1)$ does not change with Mach number, while $\Phi(\Delta p_1)$ doubles. The amplitude errors of both transport variables p_1 , u_1 also increase by this factor. The peak magnitude errors occur at $x \approx 0.8$. Here, the explicit models allow a reduction of one fourth with respect to the **IMP** results. Model **M II** only slightly improves the results of **M I**.

Altogether, Fig. 6 indicates that the newly derived models perform with higher accuracy than the implicit model **IMP**. This improvement especially concerns the axial distribution of the amplitude values, whereas the phase error of both u_1 , p_1 is almost unaffected by mean flow. Nevertheless, the TA mechanisms strongly depend on frequency. Thus, these single frequency observations do not imply a similar trend for all frequencies and mean flow conditions. Therefore, the prediction of the acoustic scattering behavior is investigated.

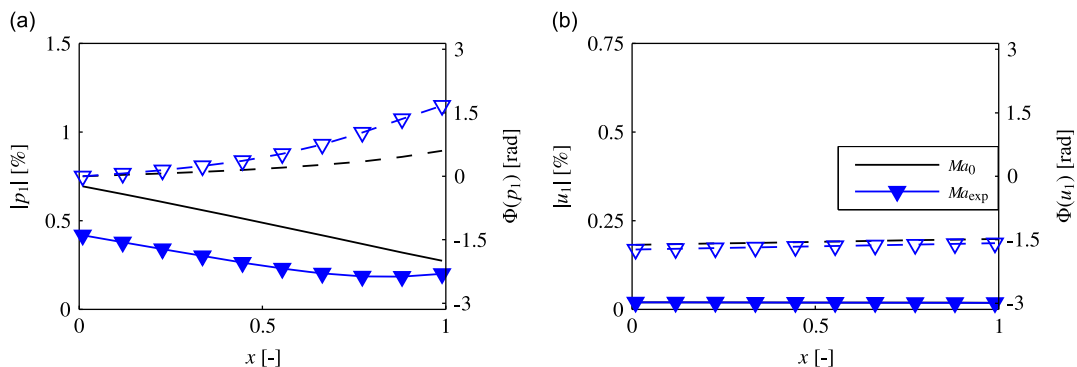


Fig. 4. Axial profiles of amplitude (solid line, filled symbols) and phase (dashed line, empty symbols) of (a) acoustic pressure p_1 and (b) acoustic velocity u_1 in a stack for two cases (Ma_0 in black and Ma_{exp} with blue triangles) extracted from CFD. While the velocity u_1 barely changes neither with space nor with mean flow, the impact of the latter is clearly visible for the pressure p_1 . (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this paper.)

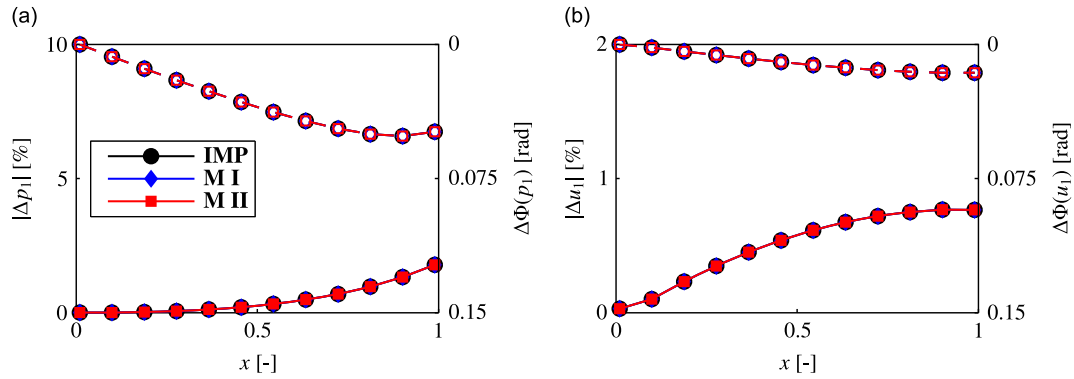


Fig. 5. Amplitude and phase error of the one-dimensional models **IMP** (●), **M I** (◆) and **M II** (■) relative to the CFD results of Ma_0 (no mean flow, $Pe=0$). The filled symbols denote gain values and phase values are depicted with empty symbols. The acoustic pressure error is depicted in (a). (b) displays the velocity error. The errors from all three 1D cases behave identically.

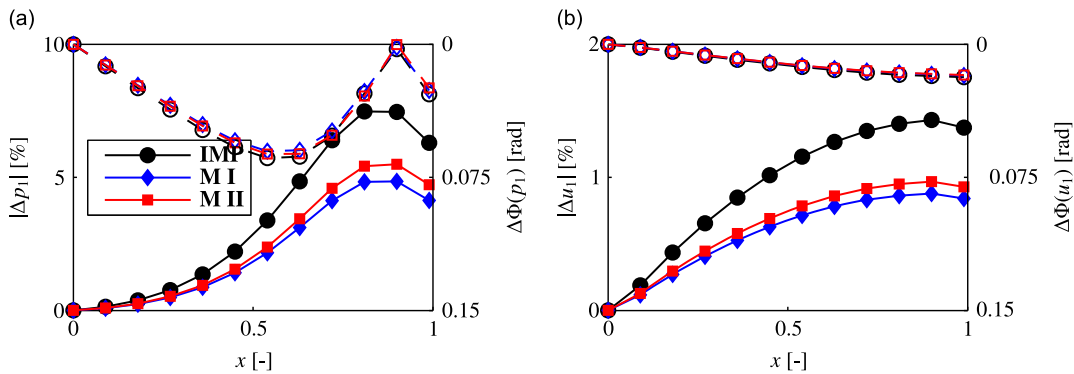


Fig. 6. Error of the one-dimensional models **IMP** (●), **M I** (◆) and **M II** (■) compared to the CFD results of Ma_{exp} (mean flow with exponential temperature profile, $Pe \approx 30$) for (a) acoustic pressure and (b) velocity. The filled symbols denote gain values and phase values are depicted with empty symbols. The amplitudes of the explicit models **M I** and **M II** deviate less from CFD than the implicit model **IMP**.

6.2. Scattering matrices of the example pore

First, the impact of mean flow and temperature field is investigated for the CFD reference data. Later, each case is compared to the results of the quasi-one-dimensional models **IMP**, **M I** and **M II**.

Fig. 7 displays the CFD/SI results of the three cases selected. The indexing of the matrix elements denotes the contribution of the incoming (first index) to the outgoing (second index) characteristic wave amplitude. Furthermore, t and r indicate, whether the wave is transmitted or reflected respectively. Solid lines denote magnitude values of the complex valued coefficients scaling with the left y-axes, while the dashed lines mark the phases belonging to the right axes.

The amplitudes of the reflection coefficients $|r|$ in Fig. 7 are relatively unaffected by mean flow and smaller than those of the transmission coefficients $|t|$. Therefore, the small discrepancies in $|r|$ are negligible and the variations in $\Phi(r)$ play a minor role.

The slope of the phase of the transmission coefficients $\Phi(t)$ is constant for varying Mach and Péclet numbers. The wiggles at $Wo > 2$ are artifacts of the system identification procedure (see [63]). General considerations reveal that these oscillations are spurious. In this region the influence of the TA effect, which steadily decreases with frequency, becomes marginal.

It was shown by Holzinger and Polifke [64] that the diagonal components of the scattering matrices control the TA energy transformation processes. Therefore, the amplitudes in the low Wo region are important for an accurate prediction of the scattering performance of a mean flow affected regenerator. The combination of a constant temperature gradient (blue curves in Fig. 7) and mean flow in the Ma_{lin} case leaves the amplitude profile of t_{ud} and t_{du} almost unaffected. There is only a slight reduction/enlargement visible for low frequencies. This corresponds to a decreasing conversion performance in such situations.

In Fig. 8 the models **IMP**, **M I** and **M II** are validated against the CFD data for $Ma_0 = 0$. All three one-dimensional models overlap perfectly. This indicates an identical implementation for this case. The coefficients r_{uu} , r_{dd} match quite well for the CFD/SI data and the one-dimensional identification methods. On the contrary, the one-dimensional solution overestimates

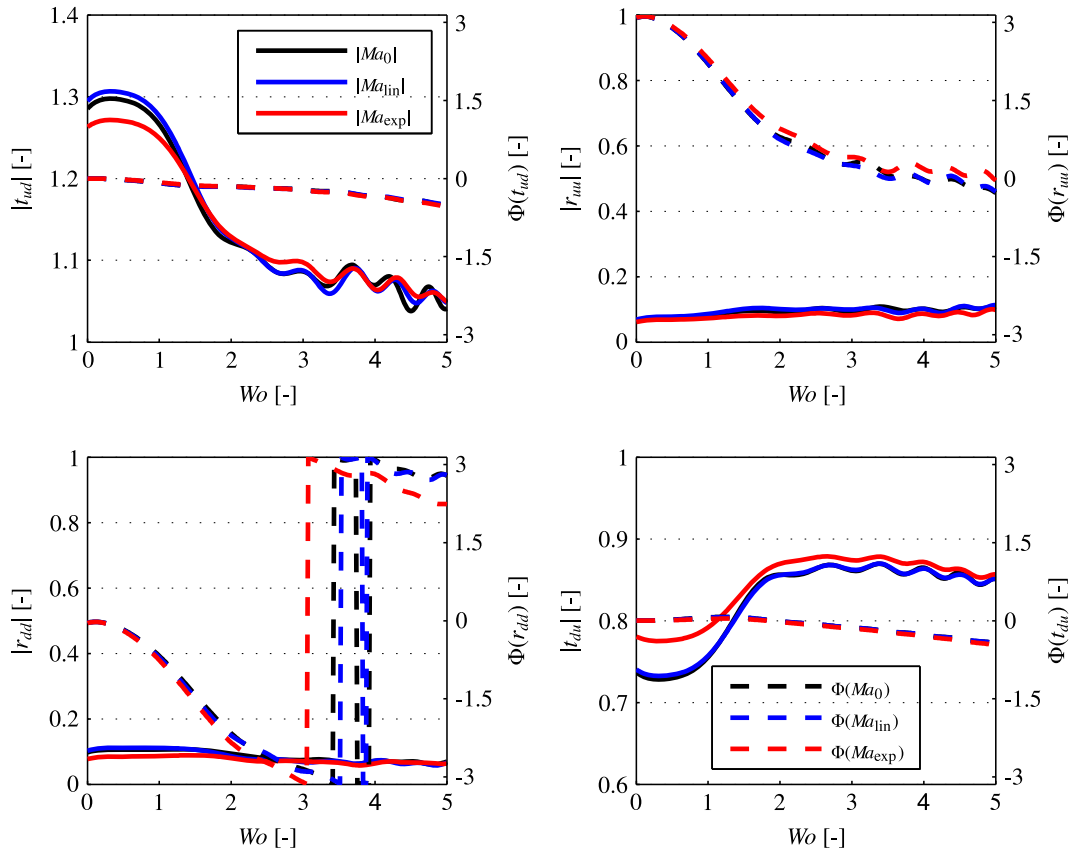


Fig. 7. Scattering matrix of a slab pore at $\Delta T = 150$ K and different Mach and Péclet numbers evaluated with the CFD/SI approach: Ma_0 (no mean flow, $Pe=0$), Ma_{lin} (mean flow with linear temperature profile, $Pe \rightarrow 0$) and Ma_{exp} (mean flow with exponential temperature profile, $Pe \approx 30$). Amplitude is depicted using a solid line and phase values are shown by dashed lines. The impact of the mean flow is strongest in both transmission coefficients of the scattering matrix at small Wo .

the TA effect for low values of Wo , although the CFD data error underestimates its contribution in this range. The sample length of 1 s only allows an accurate identification up to $Wo \gtrsim 0.5$. The CFD/SI method tends to predict the identity matrix for lower frequencies in transmission dominated acoustic elements.

Fig. 9 compares the data for Ma_{lin} , i.e. $Ma \approx 3 \times 10^{-4}$ and a vanishing Péclet number. Like the CFD data in **Fig. 7**, the explicit one-dimensional tools predict a slightly smaller TA interaction. Due to the identical mean temperature profile, the implicit model **IMP** is not at all affected by the velocity field and thus delivers results identical to Ma_0 conditions. Hence, the impact of including mean flow in the system of transport equations is small in the Ma_{lin} configuration. Thus, all models are applicable within a certain accuracy.

The good agreement of the phases is also valid for all phase values of **IMP** and CFD data in the $Pe \approx 30$ case shown in **Fig. 10**. Especially the phase of the reflected components $\Phi(r)$ agree far better in the low frequency region. However, due to the low magnitude of these coefficients, the contribution to the overall scattering behavior is negligible.

The phase differences of the 1D results in the $Wo < 1$ region of t_{ud} and t_{du} are much smaller than the latter. The **IMP** formulation predicts an amplitude profile identical to Ma_0 for both transmission coefficients. They overestimate the influence of the TA effect by a value of 0.1, which is undesirable. Although the CFD prediction attenuates the amplification of the downstream (cold to hot) traveling wave by approximately 95 percent of the Ma_0 case, **IMP** does not consider this change. In contrast to **IMP**, **M I** predicts lower contributions of the thermo-viscous acoustic boundary layers to the transmission performance than the CFD/SI. When the frequency is reduced below $Wo \approx 1.5$ the contributions strongly tend back to unity. In this range, the accordance of **M II** and the CFD simulation is very good. The curves of both identification processes almost match. Due to the steeper temperature gradient, the contributions of combined mean temperature and velocity terms mentioned for $Pe \rightarrow 0$ play a more profound role here. They capture the reduced energy transformation from heat and acoustics. It has to be stated that if Womersley numbers lower than 0.5 are considered, the solution becomes unphysical. The accumulation of numerical errors in the explicit mean velocity terms causes chaotic scattering predictions for the transmission coefficients that extend the displayed amplitude range by far [65]. This problem is also observed for **M I** although occurring at far lower frequencies.

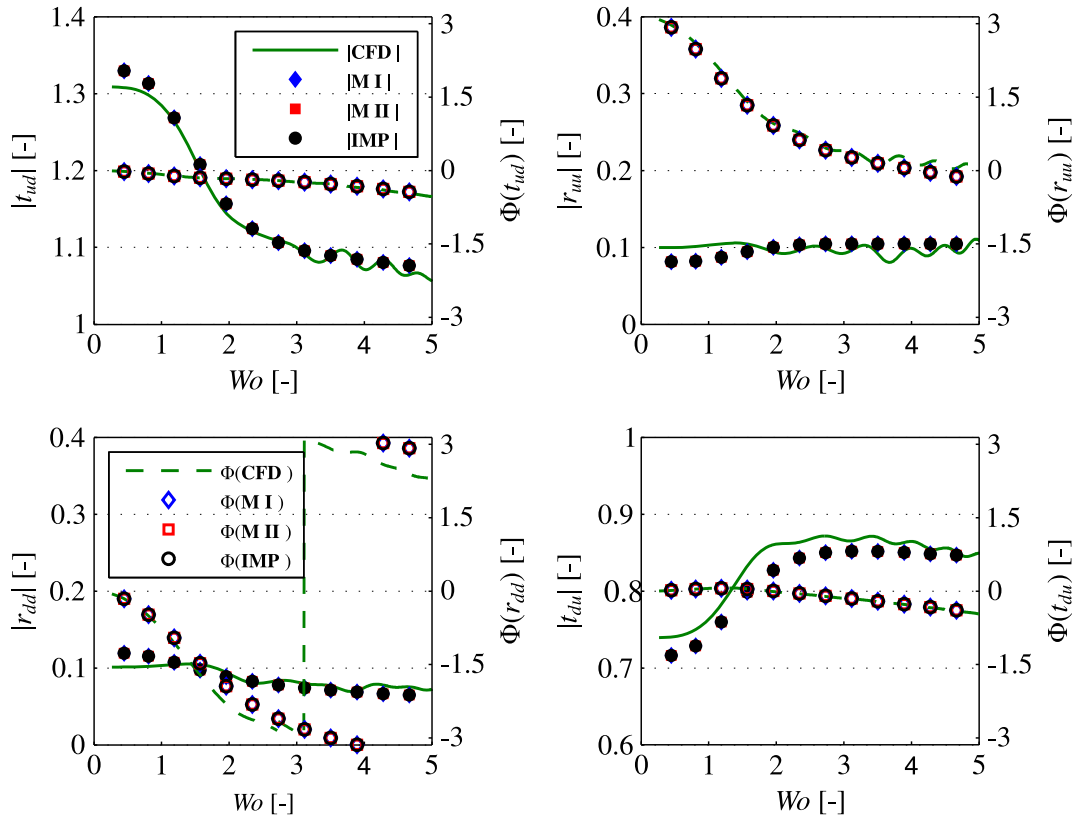


Fig. 8. Scattering matrix of a slab pore at $\Delta T = 150$ K and Ma_0 (no mean flow, $Pe=0$). Amplitude is depicted using a solid line for the CFD or filled symbols in case of the one-dimensional models. Phase values are shown by dashed lines or empty symbols respectively. All one-dimensional models **IMP** (●), **MI** (■) predict equal values for the trivial case. They match quite well with the CFD solution.

6.3. Physical explanations

Finding a physical explanation for the thermoacoustic transport behavior under mean flow conditions is not straightforward. Two ideas are presented. While the former emphasizes the physics of the problem, the latter considers the mathematical aspects of the LNSEs.

Reid [30] already proposed an explanation of the reduction in energy transmission by considering the temperature of gas parcels inside the stack. This temperature is displayed in Fig. 11 for stagnant and mean flow conditions respectively. Reid argues that the enclosed area (gray) in the $T-x$ diagram scales with the energy transported by a fluid parcel. The area in the presence of mean flow should be smaller than for stagnant conditions. Thus, the thermoacoustic effect should be less effective with mean flow, which is indeed observed for the cases considered in this paper. At lower frequencies the ratio of the number of oscillations that a parcel experiences while passing through the stack to its time of residence is smaller, thus this effect would account for the reduction of the thermoacoustic hump for low Womersley numbers.

The second, more mathematical approach is a consideration of the cross-sectional averaging functions occurring at the application of Green's function technique. The parameters α_j occurring in the differential operator $\mathcal{L}_j$ of both the axial momentum equation (26c) and the energy equation (26e) are located on the positive imaginary axis of the complex plane. The additional terms in Eqs. (32) and (33) lead to a shift of α_j away from this axis. The shape of the thermoacoustic hump in the scattering matrices correlates with the cross-sectional averaging function F_j of the pore considered [17,64]. If the argument of the function varies, its result also changes drastically. For low Wo , the Helmholtz number κ is also small. Thus, the ratio of the new upcoming terms to the first term in the differential operator becomes smaller. Hence, the change in the resulting scattering matrix becomes larger for low Wo . This argument also explains the importance of the modeled convective terms Λ_1 and Λ_2 , because they directly affect the cross-sectional shape function. They compensate the impact of the other terms such that the thermoacoustic hump is not reduced too much.

The arguments presented provide some insight into the effects of mean flow on the scattering matrices of thermoacoustic stacks at high Péclet numbers Pe . However, it is admitted that the physical effects leading to the reduction of the thermoacoustic hump at low frequencies are not completely understood. In addition, we must concede that the second argument does not imply that the thermoacoustic hump should always be reduced in the presence of mean flow.

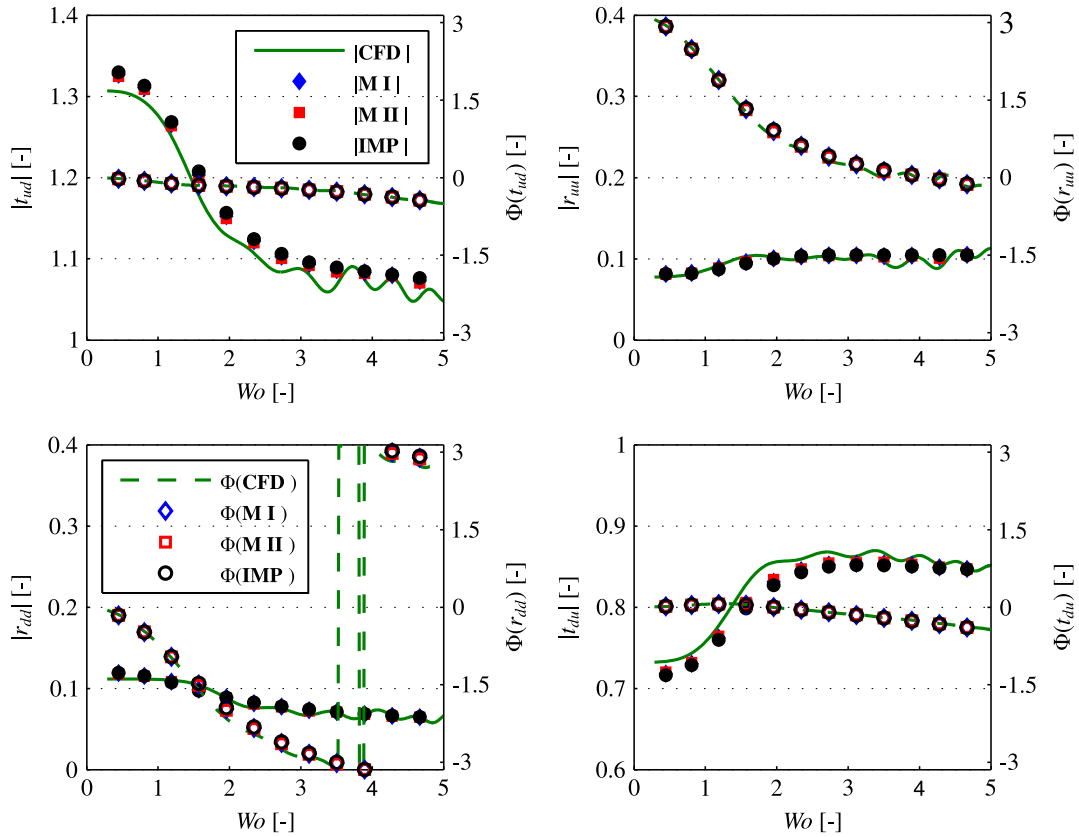


Fig. 9. Scattering matrix of a slab pore at $\Delta T = 150$ K and Ma_{lin} (mean flow with linear temperature profile, $Pe \rightarrow 0$). Amplitude is depicted using a solid line for the CFD or filled symbols in case of the one-dimensional models. Phase values are shown by dashed lines or empty symbols respectively. The one-dimensional models **IMP** (●), **M I** (◆) and **M II** (■) predict similar values for this case. The explicit models are slightly closer to the results from CFD/SL.

These arguments only provide a small insight into the physics of the thermoacoustic effect in mean flow conditions. However, the one-dimensional modeling results match very well to the CFD data. This good agreement can be seen as an a posteriori justification for correctness of the presented model. Such methods are commonly applied when new modeling approaches are developed.

7. Further improvements

The system of equations (58) derived in Section 4 used the two simplest modeling approaches, i.e. neglecting the convective terms for **M I** and inserting a spatial averaged formulation for quiescent mean conditions for **M II**. Furthermore, the mean parameters were restricted to be constant in the transversal direction. This procedure kept the expressions short, but both simplifications may be replaced by more accurate methods.

If laminar mean flow problems are considered, a parabolic velocity and temperature profile develops in the channel. Including such formulations into the LNSEs (26) leads not only into a y -dependency of the terms considered here, but also to the occurrence of additional terms that scale with transversal derivatives of the mean quantities. The first term in the differential operators $\mathcal{L}$ of Eqs. (32) and (33) then has a non-constant contribution. This differential operator leads to a combination of Weber and Gamma functions for the transversal momentum equation (26d). Additionally, the convective terms become functions of y and thus, the complexity of the system increases. Nevertheless, the derivation process still allows an analytical one-dimensional description of the acoustic transport process.

The shift from transversally averaged model approaches for Λ_i to its y -dependent equivalent may lead to more accurate results. As the GF of the problem does not change, the increase in computational costs is marginal. A third approach, which sequentially reinserts the solutions obtained by **M I** and **M II** for Λ_i also potentially improves the solutions, although the numerous terms inserted into the derivation at an early stage generate a huge number of expressions in the system matrix $\mathbf{A}(x)$.

The ideas presented lead to a more physical description of the configuration and are subject to further investigations. Nevertheless, the further improvement in accuracy will probably not justify the increasing computational costs. This prediction is supported by the fact that the enhancement reached by **M II** increased the computational time by a factor of ten.

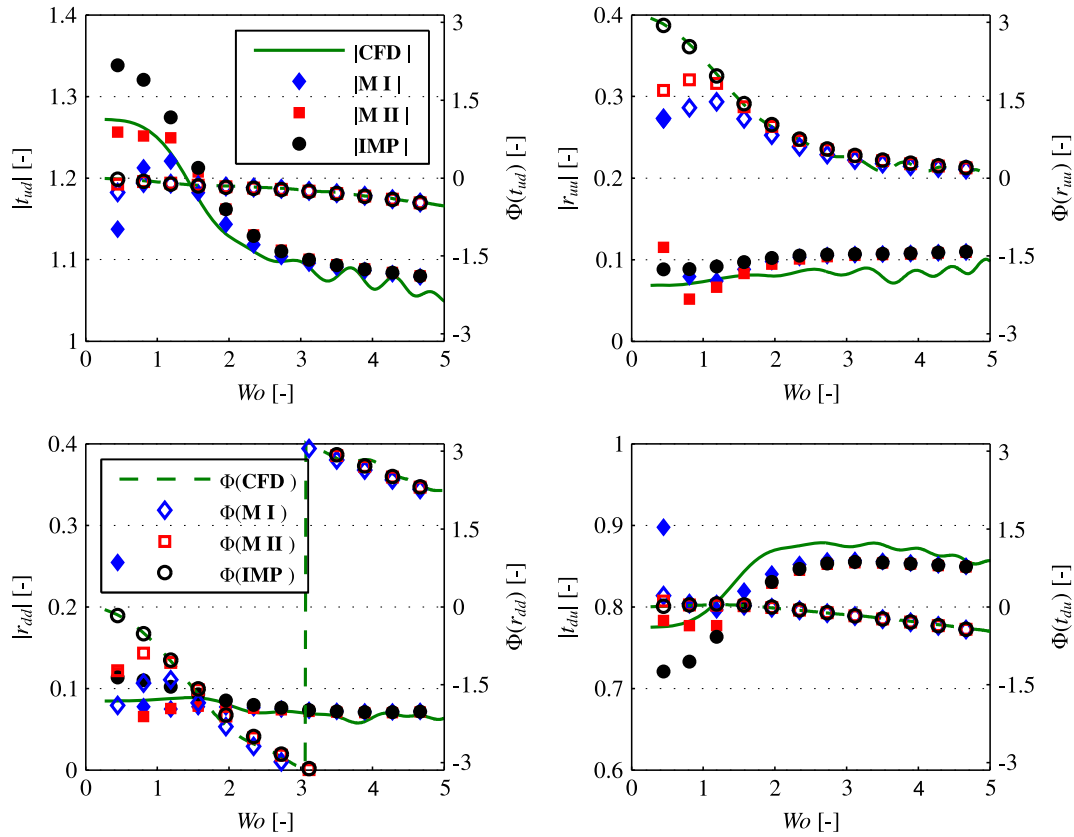


Fig. 10. Scattering matrix of a slab pore at $\Delta T = 150$ K and Ma_{exp} (mean flow with exponential temperature profile, $Pe \approx 30$). Amplitude is depicted using a solid line for the CFD or filled symbols in case of the one-dimensional models. Phase values are shown by dashed lines or empty symbols respectively. **M I** (◆) and **M II** (■) equalize the prediction quality of **IMP** (●) except for the phase of the reflection coefficients at $Wo \leq 1.5$. In all other quantities the more complex model (**M II**) matches the CFD solution better than the implicit model (**IMP**).

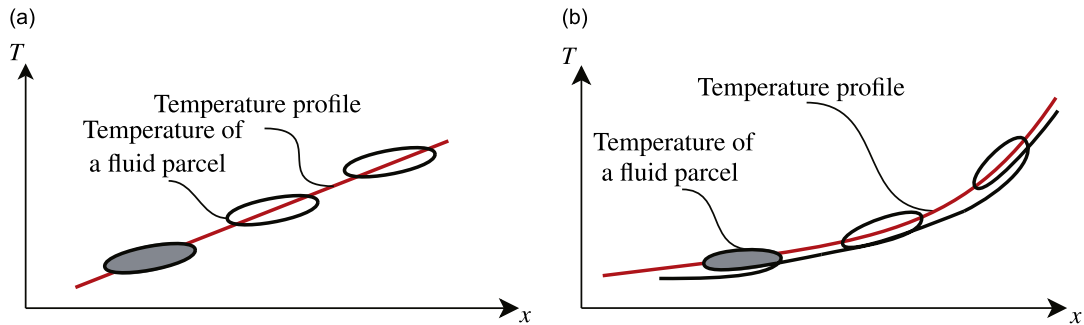


Fig. 11. Temperature of a fluid parcel considered by Reid [30]. The enclosed area (gray) of a mean flow affected parcel (b) is smaller than at stagnant mean conditions (a).

8. Conclusions

The one-dimensional explicit models derived in this study improve the accuracy of the description of the thermo-viscous interaction in narrow pores with mean flow. Earlier implicit models only incorporate the change in mean temperature distribution, which then interacts with the acoustic variables. The resulting scattering predictions of these implicit one-dimensional approaches are not affected by different, mean flow controlled temperature shapes. If mean flow is explicitly considered in the TA transport equations, the prediction of the scattering behavior is improved. An increase in complexity of the invoked closure assumptions leads to better prediction results for steep temperature profiles. Thus, the increased computational costs should be accepted in favor of a better predictability for $Wo < 2$. At very low frequencies the explicit models fail, because of the accumulation of numerical errors.

Although enhanced modeling approaches will be investigated in future, their practicability is questionable. This publication showed a good agreement of the reduced one-dimensional model **M II** to multi-dimensional CFD simulations in non-zero mean flow conditions. The expected gain in accuracy for such conditions occurring rarely in the literature is marginal. Time-averaged flow effects denoted as streaming, which are part of most TA devices, lead to non-parabolic mean flow profiles. If their magnitude is large, such conditions may necessitate a more detailed description of transversal components of the mean flow field, as well as transversally non-constant closure assumptions. If these enhancements are not sufficient, an iterative insertion of the obtained systems of equations may lead to satisfactory accuracy in the predicted results. All these improvements are accompanied by an increase in computational costs. At some point, they presumably do not justify the gain in accuracy any longer and the predictions may be carried out using coupled 1D-CFD simulations.

Concluding, the usage of the enhanced quasi-one-dimensional model derived here allows accurate predictions for mean flow affected thermoacoustic devices. The development of such apparatus beneficially applying or at least handling inherent mean flow may lead to an increase in commercial application of the thermoacoustic effect.

Appendix A. Abbreviations in formulas

A.1. Axial momentum equation

Comparing Eqs. (26c)–(34) under consideration of Eq. (32) directly leads to

$$\mathbb{X}_{p_1} = \frac{(1-\gamma)T}{i\kappa - Ma \frac{\partial T}{\partial x}} \quad (\text{A.1a})$$

$$\mathbb{X}_{A_1} = \frac{(1-\gamma)TA_1}{i\kappa - Ma \frac{\partial T}{\partial x}}. \quad (\text{A.1b})$$

A.2. Energy equation

Inserting the solution of Eq. (34) into (26e) causes the abbreviations

$$\mathbb{E}_{p_1} = \frac{T \left((-1+\gamma)\kappa - i c_p Ma \gamma \frac{\partial T}{\partial x} \right)}{c_p \left(\kappa - i Ma \frac{\partial T}{\partial x} \right)} \quad (\text{A.2a})$$

$$\mathbb{E}_{p_1, F_v} = 0 \quad (\text{A.2b})$$

$$\mathbb{E}_{p_1'} = - \frac{(Ma(-1+\gamma)T^2)}{c_p \left(i\kappa + Ma \frac{\partial T}{\partial x} \right)} \quad (\text{A.2c})$$

$$\mathbb{E}_{p_1', F_v} = - \frac{\mathbb{X}_{p_1} \frac{\partial T}{\partial x}}{i\kappa + Ma \frac{\partial T}{\partial x}} \quad (\text{A.2d})$$

$$\mathbb{E}_{A_2} = \frac{i(-1+\gamma)TA_2}{c_p \left(\kappa - i Ma \frac{\partial T}{\partial x} \right)} \quad (\text{A.2e})$$

$$\mathbb{E}_{A_1, F_v} = \frac{-\mathbb{X}_{A_1} \frac{\partial T}{\partial x}}{i\kappa + Ma \frac{\partial T}{\partial x}}. \quad (\text{A.2f})$$

Here, the terms $\mathbb{E}_{A_2}$ and $\mathbb{E}_{A_1, F_v}$ vanish for **M I**. After combining both energy equations (26e) and (26f) the new occurring variables are

$$\mathbb{T}_{p_1} = \left(\mathfrak{P}r \mathbb{E}_{p_1} + \mathbb{E}_{p_1, F_v} \left(-1 + \frac{f_{K, \nu}}{f_K} \right) \right) \mathbb{E}_{\epsilon_S} \quad (\text{A.3a})$$

$$\mathbb{T}_{p_1'} = \left(\mathfrak{P}r \mathbb{E}_{p_1'} + \mathbb{E}_{p_1', F_v} \left(-1 + \frac{f_{K, \nu}}{f_K} \right) \right) \mathbb{E}_{\epsilon_S} \quad (\text{A.3b})$$

$$\mathbb{T}_{A_1} = -(\mathbb{E}_{A_1, F_\nu} \mathbb{E}_{\epsilon_S}) + \frac{(\mathfrak{P}r \mathbb{E}_{A_2} f_K + \mathbb{E}_{A_1, F_\nu} f_{K,\nu}) \mathbb{E}_{\epsilon_S}}{f_K} \quad (\text{A.3c})$$

$$\mathbb{T}_{p_1, F_K} = \frac{\mathbb{E}_{p_1, F_\nu} (f_K - f_{K,\nu}) \mathbb{E}_{\epsilon_S} + \mathbb{E}_{p_1} f_K (1 - \mathfrak{P}r \mathbb{E}_{\epsilon_S})}{f_K} \quad (\text{A.3d})$$

$$\mathbb{T}_{p_1', F_K} = \frac{\mathbb{E}_{p_1', F_\nu} (f_K - f_{K,\nu}) \mathbb{E}_{\epsilon_S} + \mathbb{E}_{p_1'} f_K (1 - \mathfrak{P}r \mathbb{E}_{\epsilon_S})}{f_K} \quad (\text{A.3e})$$

$$\mathbb{T}_{A_2 F_K} = \frac{\mathbb{E}_{A_1, F_\nu} (f_K - f_{K,\nu}) \mathbb{E}_{\epsilon_S}}{f_K} + \mathbb{E}_{A_2} (1 - \mathfrak{P}r \mathbb{E}_{\epsilon_S}). \quad (\text{A.3f})$$

Again (A.3c) and (A.3f) vanish for the simpler model.

A.3. Gas law

The first insertion process of the resulting energy (53) and momentum equation (34) into the gas law (26a) yields two new parameters

$$\mathbb{G}_{p_1} = \frac{\gamma}{(-1 + \gamma)T} \quad (\text{A.4a})$$

$$\mathbb{G}_{T_1} = \frac{-1}{(-1 + \gamma)T^2}, \quad (\text{A.4b})$$

which are combined to

$$\overline{\mathbb{G}}_{p_1} = \mathbb{G}_{p_1} + (\mathbb{E}_{p_1, F_\nu} + \mathbb{T}_{p_1} + \mathbb{T}_{p_1, F_K} - \mathbb{T}_{p_1, F_K} f_K - \mathbb{E}_{p_1, F_\nu} f_{K,\nu}) \mathbb{G}_{T_1} \quad (\text{A.5a})$$

$$\overline{\mathbb{G}}_{p_1'} = (\mathbb{E}_{p_1', F_\nu} + \mathbb{T}_{p_1'} + \mathbb{T}_{p_1', F_K} - \mathbb{T}_{p_1', F_K} f_K - \mathbb{E}_{p_1', F_\nu} f_{K,\nu}) \mathbb{G}_{T_1} \quad (\text{A.5b})$$

$$\overline{\mathbb{G}}_{A_1, A_2} = (\mathbb{T}_{A_1} + \mathbb{T}_{A_2 F_K} + \mathbb{E}_{A_1, F_\nu} - \mathbb{T}_{A_2 F_K} f_K - \mathbb{E}_{A_1, F_\nu} f_{K,\nu}) \mathbb{G}_{T_1} \quad (\text{A.5c})$$

in the averaging process. Again the last term becomes zero for **M I**.

A.4. Conservation of mass

Finally, all derived formulas are inserted in the conservation of mass (26b) and the intermediate values

$$\overline{\overline{\mathbb{K}}}_{\langle u_1 \rangle} = \frac{-i \frac{\partial T}{\partial x}}{(-1 + \gamma)T^2 \left(\kappa + iMa \frac{\partial T}{\partial x} \right)} \quad (\text{A.6a})$$

$$\overline{\overline{\mathbb{K}}}_{\langle u_1' \rangle} = \frac{i}{\left((-1 + \gamma)T \left(\kappa + iMa \frac{\partial T}{\partial x} \right) \right)} \quad (\text{A.6b})$$

$$\overline{\overline{\mathbb{K}}}_{\langle e_1' \rangle} = \frac{-i(-MaT^3) + Ma\gamma T^3}{(-1 + \gamma)T^2 \left(\kappa + iMa \frac{\partial T}{\partial x} \right)} \quad (\text{A.6c})$$

result in the coefficients

$$\overline{\mathbb{K}}_{u_1'} = - \frac{(f_\nu - 1)_{\langle u_1 \rangle} \mathbb{X}_{p_1}}{(f_\nu - 1) \overline{\overline{\mathbb{K}}}_{\langle u_1' \rangle} \mathbb{X}_{p_1} - \overline{\mathbb{G}}_{p_1'} \overline{\overline{\mathbb{K}}}_{\langle e_1' \rangle}} \quad (\text{A.7a})$$

$$\overline{\mathbb{K}}_{A_1, A_2} = - \frac{\overline{\mathbb{G}}_{p_1'} \overline{\overline{\mathbb{K}}}_{\langle e_1' \rangle} \mathbb{X}_{A_1} f_\nu'}{(f_\nu - 1) \overline{\overline{\mathbb{K}}}_{\langle u_1' \rangle} \mathbb{X}_{p_1} - \overline{\mathbb{G}}_{p_1'} \overline{\overline{\mathbb{K}}}_{\langle e_1' \rangle}} \frac{\overline{\mathbb{G}}_{A_1, A_2} \left(\overline{\mathbb{G}}_{A_1, A_2} - \overline{\overline{\mathbb{K}}}_{\langle e_1' \rangle} \frac{\partial \mathbb{X}_{A_1, A_2}}{\partial x} \right) + \overline{\mathbb{G}}_{p_1'} \overline{\overline{\mathbb{K}}}_{\langle e_1' \rangle} \frac{\partial \mathbb{X}_{A_1}}{\partial x}}{(f_\nu - 1) \overline{\overline{\mathbb{K}}}_{\langle u_1' \rangle} \mathbb{X}_{p_1} - \overline{\mathbb{G}}_{p_1'} \overline{\overline{\mathbb{K}}}_{\langle e_1' \rangle}} \quad (\text{A.7b})$$

$$\overline{\overline{\mathbb{K}}}_{p_1'} = \frac{\overline{\mathbb{G}}_{p_1'} \left[(f_\nu - 1) \left(\mathbb{X}_{p_1} + \overline{\overline{\mathbb{K}}}_{\langle e_1' \rangle} \frac{\partial \mathbb{X}_{p_1}}{\partial x} \right) + \mathbb{X}_{p_1} \overline{\overline{\mathbb{K}}}_{\langle e_1' \rangle} \frac{\partial f_\nu}{\partial x} \right]}{(f_\nu - 1) \overline{\overline{\mathbb{K}}}_{\langle u_1' \rangle} \mathbb{X}_{p_1} - \overline{\mathbb{G}}_{p_1'} \overline{\overline{\mathbb{K}}}_{\langle e_1' \rangle}}$$

$$-\frac{(f_\nu - 1)\overline{\overline{\mathbb{K}}}_{(q_1')} \mathbb{X}_{p_1} (\overline{\mathbb{G}}_{p_1} + \frac{\partial \overline{\mathbb{G}}_{p_1'}}{\partial x})}{(f_\nu - 1)\overline{\overline{\mathbb{K}}}_{(u_1')} \mathbb{X}_{p_1} - \overline{\mathbb{G}}_{p_1'} \overline{\overline{\mathbb{K}}}_{(q_1')}} \quad (\text{A.7c})$$

$$\overline{\overline{\mathbb{K}}}_{p_1} = \frac{(f_\nu - 1)\mathbb{X}_{p_1} (\overline{\mathbb{G}}_{p_1} - \overline{\overline{\mathbb{K}}}_{(q_1')} \frac{\partial \overline{\mathbb{G}}_{p_1}}{\partial x})}{(f_\nu - 1)\overline{\overline{\mathbb{K}}}_{(u_1')} \mathbb{X}_{p_1} - \overline{\mathbb{G}}_{p_1'} \overline{\overline{\mathbb{K}}}_{(q_1')}} \quad (\text{A.7d})$$

for the second transport PDE of the acoustic variables containing the term $\overline{\overline{\mathbb{K}}}_{\Lambda_1, \Lambda_2}$ which contains the terms differing in the model derivations.

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